

Joint CFO-Channel Estimation over CFO-Coherent SS Burst Sets for Low-Altitude Radio Mapping

Bowen Li, Haotian Zhang, Mu Jia, Junting Chen, Nikolaos Pappas

Abstract—Extending terrestrial networks into low-altitude airspace is a practical way to support aerial services, and accurate low-altitude radio maps are essential for characterizing terrestrial base station (BS) coverage and guiding system design. This work targets per-cell per-beam radio mapping from 5G new radio (NR) synchronization signal (SS) burst sets. Conventional processing treats interference as noise and focuses on the strongest link, which is insufficient to comprehensive awareness of the radio environment and ineffective in dense multi-cell low-altitude scenarios. We propose a successive waveform reconstruction and cancellation framework that iteratively estimates, reconstructs, and subtracts the SSs of stronger BSs, thereby enabling reliable detection and estimation of ultra-weak signals. To support this, we introduce the notion of a carrier frequency offset (CFO)-coherent block within which a common-CFO/per-synchronization signal block (SSB)-channel model holds and design a joint CFO-channel estimator that coherently aggregates multiple SSBs within each CFO-coherent block. We further derive closed-form scaling laws that relate estimation accuracy to unmanned aerial vehicle (UAV) speed, motion geometry, burst periodicity, and the length of the CFO-coherent block. Simulations show that the proposed framework can detect and estimate SSs at signal-to-interference-and-noise ratio (SINR) levels down to -30 dB. Field tests at 150 m altitude demonstrate per-beam coverage maps for more than ten overlapping BSs and reveal that, despite strong received power, the measured SINR rarely exceeds 10 dB, underscoring the need for careful interference management in low-altitude airspace.

Index Terms—Low-altitude radio map, CFO-coherent block, 5G NR SS burst, multi-cell interference, joint CFO-channel estimation.

I. INTRODUCTION

The low-altitude economy, encompassing activities such as automated logistics and real-time infrastructure inspection within the airspace from ground level up to 3000 meters, has expanded rapidly, creating a critical demand for low-altitude wireless communications [1], [2]. Extending existing terrestrial 5G networks to low-altitude airspace offers the most pragmatic solution. This is primarily because: i) 5G networks are already

densely deployed, providing a widespread and readily available infrastructure; ii) 5G base stations (BSs) employ massive antenna arrays, which offer significant potential for spatial multiplexing and precise 3D beamforming; and iii) the low-altitude environment features largely unobstructed propagation paths, leading to favorable line-of-sight (LOS) conditions.

However, the terrestrial BS antennas are optimized for ground users, with their main lobes intentionally tilted downwards and only unintended, irregular side lobes radiating towards the sky [3]–[5]. Consequently, aerial coverage is a complex superposition of direct signals from these side lobes and multi-path signals from main lobes reflecting off the ground. This creates a highly heterogeneous signal landscape that defies conventional, monotonic decay models, making it extremely difficult to predict. Therefore, to effectively utilize terrestrial networks for aerial services, such as predictive communications [6], flight trajectories design [7], or BS beams optimization for aerial users [8], it is first imperative to chart the actual radio environment. This establishes an urgent need for accurate *low-altitude radio maps* to truly understand the coverage provided by terrestrial BSs in the low-altitude airspace.

State-of-the-art radio mapping [9] largely focuses on data-level techniques, such as matrix completion [10] and learning-based prediction [11], or model-driven approaches like digital twins [12] and ray-tracing [13]. The former class of methods infers a complete map from a sparse set of trusted measurements, while the latter simulates the radio environment based on detailed geographical models. Our focus, however, is not at the data or model level, but at the physical layer, where we tackle the fundamental challenge of constructing the trusted, ground-truth measurements from raw 5G signals, thereby providing the foundational technology upon which both data- and model-driven techniques depend.

The 5G new radio (NR) synchronization signal (SS) burst structure inherently facilitates per-cell and per-beam radio map construction [14]–[16]. Specifically, in each synchronization signal block (SSB), the primary and secondary synchronization signals (PSS/SSS) encapsulate the physical cell identity (PCI), while the demodulation reference signal (DMRS) conveys beam index information. Collectively, these SSs enable precise channel estimation and beam-level identification. However, conventional estimation and detection algorithms predominantly prioritize the extraction of the strongest signal component [17]–[21]. This paradigm is insufficient for radio map construction, which mandates a holistic awareness of the comprehensive radio environment, including the weak signal components, rather than the isolation of a singular dominant

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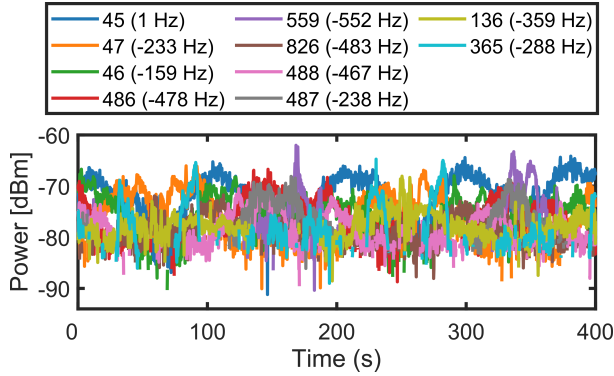


Figure 1. The challenge of low-altitude measurement: the decorrelation of the massive amount of BS signals with diverse frequency offsets.

link.

Furthermore, the practical construction of radio maps in low-altitude environments is confronted by two critical challenges, as illustrated in Fig. 1:

- *Strong interference*: A large number of terrestrial BSs become LOS in the sky, and their SS signals overlap in both time and frequency with comparable received power. As a result, the SS of interest may experience signal-to-interference-and-noise ratio (SINR) values as low as -30 dB, far below the operating range of conventional detection and estimation algorithms.
- *Diverse carrier frequency offsets (CFOs)*: Different BSs contribute distinct oscillator offsets, while the motion geometry of airspace sampling platforms (e.g., unmanned aerial vehicle (UAV)) induces time-varying Doppler shifts. The resulting superposition of multiple CFOs severely degrades the correlation with the SSs, hindering reliable detection and accurate parameter estimation.

These challenges inevitably result in missed detection, false alarms, and channel estimation bias. Although successive interference cancellation (SIC) [22], [23] can, in principle, mitigate compounded interference, under strong interference conditions, even small errors in the dominant signal's CFO and channel estimates leave residuals whose power remains higher than that of the weaker signals, rendering them effectively undetectable and yielding severely biased estimates. Early works exploited the cyclic prefix (CP) of the orthogonal frequency division multiplexing (OFDM) symbols or dedicated synchronization sequences to jointly estimate the channel and CFO in single-transmitter systems [24], [25]. Subsequent studies have extended these frameworks to multi-transmitter architectures, such as multi-point relay systems [26], coordinated multi-cell systems [27], and distributed antenna systems [28], and design joint estimation algorithms to cope with severe co-channel interference and multiple frequency offsets [29]–[35]. To summarize, existing approaches for joint CFO-channel estimation may only work in the scenario with a small number of weak interferers (e.g., 3–4 interferers). For an extremely low SINR, a typical case in low-altitude radio map surveying, the conventional CFO estimation may fail and the corresponding channel estimation may collapse.

In this paper, we process multiple 5G NR SS burst sets for

joint CFO-channel estimation, where each SS burst set contains a few SSBs precoded by dedicated and fixed beamformers. We propose detecting ultra-low signals by successively estimating and reconstructing signals from stronger BSs and subtracting them from the received waveform. Such an approach requires an accurate estimation of the CFOs, which are estimated by robust multi-burst processing across SS burst sets. Specifically, this paper makes the following contributions.

- We propose a successive waveform reconstruction and cancellation framework that successively estimates and reconstructs the synchronization signals of stronger BSs and subtracts them from the received waveform. This suppresses multi-cell LOS interference and enables the reliable detection and estimation of very weak synchronization signals that would otherwise be obscured.
- We introduce the notion of a CFO-coherent temporal block and analytically characterize its duration in terms of UAV-BS relative motion and Doppler dynamics. Exploiting this CFO coherence, we develop a cross-burst joint CFO-channel estimation framework that assumes CFO constancy across the block and channel constancy within each SS burst, enabling coherent aggregation of multiple bursts for substantially improved CFO and channel estimation of weak signals.
- We analytically derive how CFO and channel estimation accuracy scale with UAV speed, motion geometry, SS burst periodicity, and the length of the CFO-coherent block, yielding physically grounded models that quantify the fundamental accuracy-motion trade-offs in UAV-based measurement campaigns.
- We numerically show that the proposed method can reliably detect, estimate, and reconstruct synchronization signals at SINR as low as -30 dB under unknown CFO. Our field tests further demonstrate the construction of per-beam coverage maps for more than 10 overlapping BSs at an altitude of 150 meters. It is found that despite strong received power (≥ -65 dBm) across nearly the entire flight area, the measured SINR rarely exceeds 10 dB, highlighting the need for careful interference management in low-altitude airspace.

The rest of the paper is organized as follows. Section II introduces the preliminaries and discusses the challenges of low-altitude radio mapping. Section III introduces the CFO-coherent block and derives the motion conditions required to guarantee CFO coherence. Section IV develops the multi-SSB processing for joint CFO and channel estimation. Section V presents the proposed cross-burst detection and estimation framework. Numerical and experimental results are presented in Section VI, and conclusions are given in Section VII.

II. PRELIMINARIES AND CHALLENGES

This section introduces the 5G NR SS burst structure and explains why it is suitable for per-cell, per-beam radio map construction in low-altitude environments, then summarizes classical detection and estimation pipelines and illustrates their lack of robustness in low-altitude environments.

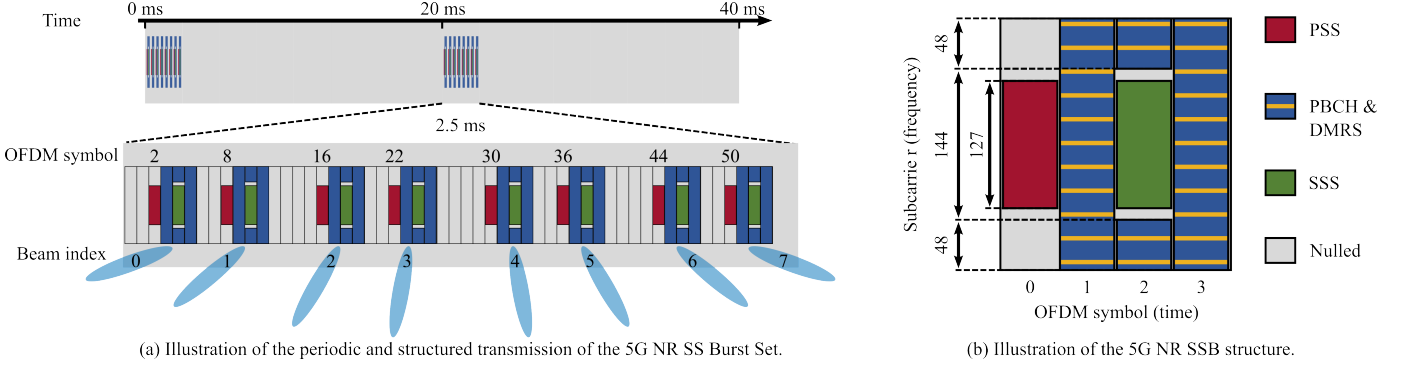


Figure 2. Illustration of the 5G NR SS burst structure. (a) Periodic transmission of an SS-burst set with a default periodicity of 20 ms in 3GPP case C, where up to eight SSBs are transmitted within a 2.5 ms window at predetermined OFDM symbols and beams. (b) Time-frequency structure of a single SSB spanning four OFDM symbols and 240 subcarriers, composed of PSS, SSS, and PBCH with comb-type DMRS.

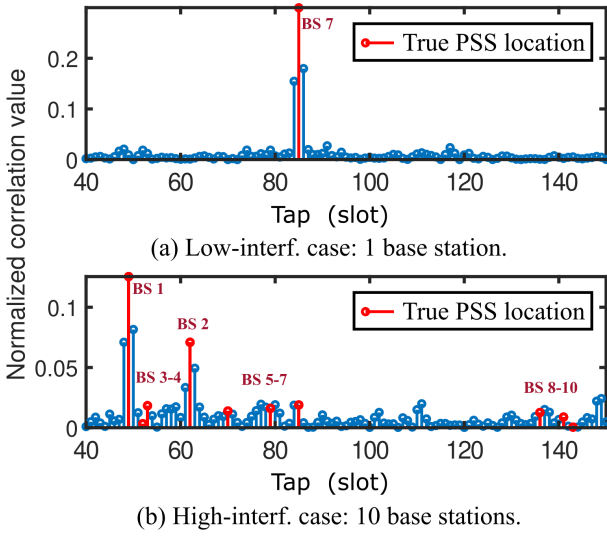


Figure 3. Impact of interference on PSS-based timing synchronization and detection. (a) In the low-interference (single-cell) case, the correlation peak provides an unambiguous timing estimate and PSS detection. (b) In the high-interference (10-cell) case, the correlation profile is severely corrupted: strong peaks from interfering cells create significant ambiguity, while the true correlation peaks are either severely attenuated or completely suppressed.

A. 5G NR SS Burst Set Structure

The 5G NR physical layer adopts a well-defined synchronization and broadcast hierarchy for robust initial access. As shown in Fig. 2, an SS-burst set is periodically transmitted with a typical periodicity of 20 ms and occupies at most 5 ms, enabling beam sweeping via multiple SSBs on distinct transmit beams [14]. The SSB's position is determined by the SSB case (A-E) and the SSB index within a burst set [15]. For example, in case C, up to eight SSBs may be transmitted within a short window (about 2.5 ms) to facilitate spatial coverage (beam pattern), as shown in Fig. 2 (a).

Each SSB comprises the PSS, SSS, and PBCH with DMRS [16], as shown in Fig. 2 (b). The PSS and SSS sequences are deterministically derived from the PCI N_{ID}^{cell} : PSS is a length-127 m-sequence uniquely determined by $N_{ID}^{(2)} = N_{ID}^{cell} \bmod 3$; the SSS is a length-127 m-sequence determined by $(N_{ID}^{(1)}, N_{ID}^{(2)})$, where $N_{ID}^{(1)} = \lfloor N_{ID}^{cell} / 3 \rfloor$. The position and

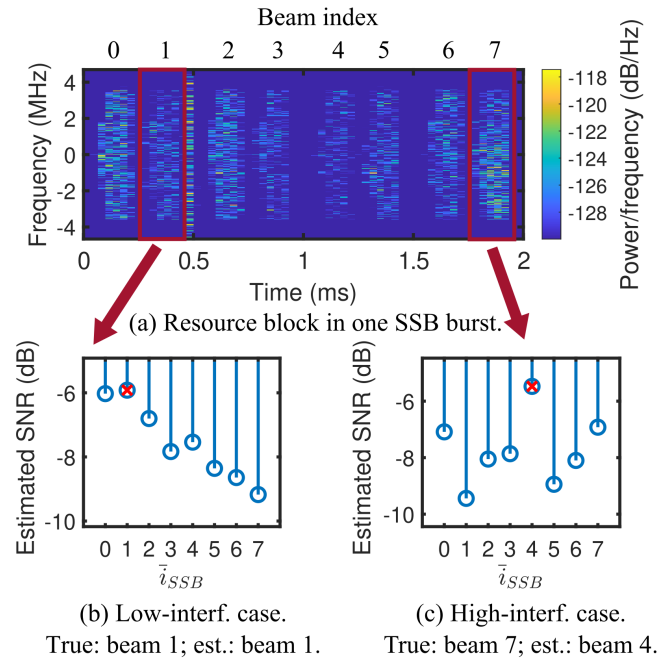


Figure 4. Impact of interference on DMRS-based SSB index identification. (a) analyzes an empirically measured SS burst set. (b) For a low-interference beam, the estimated SNR profile correctly identifies the true SSB index (1). (c) However, for a high-interference beam, co-channel interference from other cells corrupts the SNR estimation, leading to the erroneous identification of index 4 instead of the true index 7.

content of PBCH DMRS depends on the PCI and the SSB index.

The structure of the SS burst set enables low-altitude radio mapping that is feasible, stable, and resilient to interference. Fine-grained per-cell per-beam mapping follows from the deterministic embedding of the PCI in the PSS/SSS and of the SSB index in the PBCH DMRS. Stability and on-demand availability result from the rapid and periodic SSB transmissions. Moreover, the SS burst structure itself facilitates accurate detection and estimation through (i) cross-beam CFO coherence over the short burst duration (≤ 5 ms) and (ii) predetermined SSB positions and patterns, which are jointly exploited in Sections IV and V to achieve high-accuracy

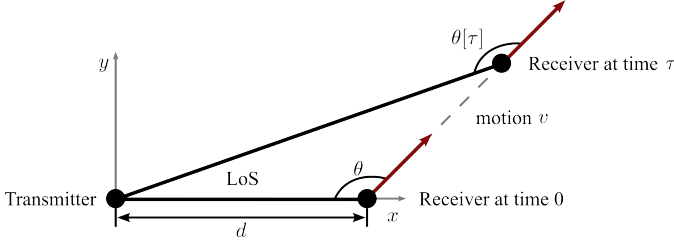


Figure 5. Sampling-platform motion model. The x-y plane is defined by the transmitter-receiver-motion geometry at time 0, with the LOS direction aligned to the positive x-axis.

processing under multi-cell interference.

B. Conventional Pipelines and Challenges

The conventional pipelines employ a sequential, multi-stage processing chain to extract synchronization parameters and channel state information [17]–[21]. The process initiates with PSS detection in the time domain, where the timing offset, CFO, and partial cell identity ($N_{ID}^{(2)}$) are jointly estimated by maximizing the correlation between the received waveform and local PSS replicas. Following time-frequency compensation, SSS detection is conducted in the frequency domain to resolve the full PCI (N_{ID}^{cell}). Finally, PBCH DMRS processing identifies the specific beam index (i_{SSB}) by matching the received signal in the frequency domain with the candidate reference signals.

However, the above pipeline only works for the case of detecting the strongest signal while treating all the interference as noise. As illustrated in Fig. 3, when the reception contains signals from 10 BSs, the PSS detection fails for BSs 3 to 10 although their SSs are nearly orthogonal. Likewise, the detection of the SSB beam index is not possible either under strong interference, as illustrated in Fig. 4. Moreover, owing to the sequential nature of the conventional pipeline, errors in time–frequency offset and root selection propagate downstream and ultimately corrupt the estimated channel gains.

III. CFO-COHERENT BLOCK

This section introduces the notion of a *CFO-coherent block* and derives sufficient conditions under which practical systems can guarantee CFO coherence over the observation interval.

Intuitively, a CFO-coherent block is a time window during which the CFO drift is small enough that all SSBs within the window share, up to a negligible error, a common CFO. Formally, let $f(t)$ denote the instantaneous CFO and consider an interval $[t, t + \tau]$. Define the CFO drift over this interval as

$$\Delta f \triangleq \max_{t_1, t_2 \in [t, t + \tau]} |f(t_1) - f(t_2)|.$$

For a prescribed coherence limit $\delta_{\max} > 0$, we say that $[t, t + \tau]$ is a *CFO-coherent block* if $\Delta f \leq \delta_{\max}$.

In 5G NR, all SSBs in a burst set are transmitted within a short window $T_{\text{burst}} < 2.5$ ms; under typical oscillator stability, the resulting CFO variation over this interval is negligible, so the entire burst set can be regarded as a single CFO-coherent block and processed coherently across SSBs. However, the

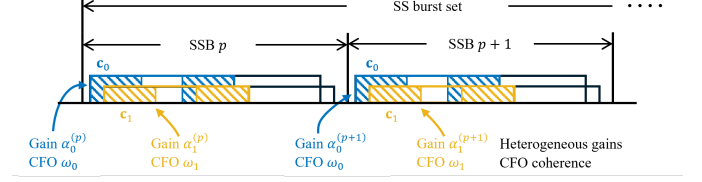


Figure 6. SS-burst-set model under multi-cell interference: the per-SSB channel gains are heterogeneous across beams, while the CFO remains coherent (common) over all SSBs within each CFO-coherent block.

sampling platform (e.g., a UAV-mounted receiver) is typically moving. Its motion induces a time-varying Doppler shift and thus a drift in the effective CFO, which, if too large, destroys cross-SSB CFO coherence. Next, we relate the CFO drift to the platform motion and derive a condition under which the entire burst set remains within a single CFO-coherent block.

Denote the sampling platform by its speed v and motion direction θ in the LOS-based coordinate system shown in Fig. 5, where θ is the angle between the velocity vector and the LOS, and let d be the transmitter-receiver distance at a reference time.

Lemma 1. (*Motion-induced CFO drift.*) *The CFO drift for a receiver moving with speed-direction pair (v, θ) at relative distance d from the transmitter is*

$$\Delta f = \frac{v}{\lambda_c} \left(\frac{d \cos \theta + v\tau}{\sqrt{d^2 + 2dv\tau \cos \theta + v^2\tau^2}} - \cos \theta \right)$$

which is approximated to $\Delta f \approx \frac{\tau v^2}{\lambda_c d} \sin^2 \theta$ with approximation error of order $\mathcal{O}(v^2\tau^2/d^2)$, where λ_c is carrier wavelength.

Proof. See Appendix A. $\square$

Lemma 1 shows that, over a short time interval, the CFO drift grows linearly with the interval length, quadratically with the platform speed, and is inversely proportional to the distance. Given a prescribed coherence limit on the CFO drift $\delta_{\max}$, the motion of sampling platform should satisfy the following condition.

Theorem 1. (*Speed condition for CFO coherence.*) *Given the coherence limit on the CFO drift $\delta_{\max}$, the platform speed must satisfy*

$$v \leq \sqrt{\frac{\delta_{\max} \lambda_c d}{\tau \sin^2 \theta}}.$$

Taking, for illustration, $\delta_{\max} = 0.1$ Hz, $\lambda_c = 0.1$ m ($f_c = 3$ GHz), $d = 100$ m, and one SS-burst duration $\tau = 0.0025$ s, the sufficient condition for CFO coherence is $v \leq \sqrt{400/\sin^2 \theta}$. Thus, under worst-case tangential motion ($\theta = \pi/2$), CFO coherence over one burst set requires $v \leq 20$ m/s.

IV. MULTI-SSB PROCESSING FOR JOINT CFO-CHANNEL ESTIMATION

Consider an SS-burst set consisting of P SSBs, indexed by $p \in \mathcal{P} \triangleq \{0, 1, \dots, P-1\}$, where each SSB contains two

synchronization sequences (PSS and SSS)¹ as shown in Fig. 6. Each received SS is a superposition of signals from K BSs, indexed by $k \in \mathcal{K} \triangleq \{0, 1, \dots, K-1\}$. For a given BS k , all SSBs within one CFO-coherent block share a common CFO, whereas each SSB p experiences its own complex channel gain.

A. Waveform Model

Let $\mathcal{C}$ denote the set of all length- N synchronization sequences. The SS transmitted by the k th BS can be expressed as

$$\mathbf{c}_k = \mathbf{c}_{k,0} \oplus \mathbf{0} \oplus \mu_k \mathbf{c}_{k,1} \quad (1)$$

where $\mathbf{c}_{k,0}, \mathbf{c}_{k,1} \in \mathcal{C}$ are the selected synchronization sequences; individual sequences may be reused across different BS, but the ordered pair $(\mathbf{c}_{k,0}, \mathbf{c}_{k,1})$, and thus the SS $\mathbf{c}_k$ is unique to BS k ; $\mu_k \in \mathbb{R}^+$ is a scaling factor that represents the power difference between the two synchronization sequences; and $\mathbf{0}$ is a zero-padding vector of fixed length τ_0 , corresponding to the timing offset between the two synchronization signals.

The cross-correlation between two synchronization sequences $\mathbf{c}_k, \mathbf{c}_l \in \mathcal{C}$ with timing offset τ and frequency offset ω is given by

$$r_{k,l}(\tau, \omega) \triangleq \frac{1}{N} \sum_{m=0}^{N-1} c_k[m] c_l^*[m-\tau] e^{-j\omega m}. \quad (2)$$

Because synchronization sequences are not perfectly orthogonal, we model the cross-correlation for any mismatch ($k \neq l$) or nonzero delay ($\tau \neq 0$) as zero-mean circular complex Gaussian with a variance that decays with the sequence length N . For the matched case ($k = l$ and $\tau = 0$), the statistic reduces to the sequence's autocorrelation $r_k(\omega)$. As a result, the cross-correlation can be expressed as

$$r_{k,l}(\tau, \omega) = \begin{cases} r_k(\omega) & k = l, \tau = 0 \\ \mathcal{CN}(0, \sigma_c^2/N) & \text{otherwise} \end{cases} \quad (3)$$

where σ_c^2 captures sequence-family sidelobes and implementation impairments (e.g., filtering, residual multipath), and the magnitude of autocorrelation $|r_k(\omega)|$ follows a Dirichlet profile; $|r_k(0)| = 1$ and although oscillatory, its overall trend decreases as $|\omega|$ increases, e.g., for Zadoff-Chu sequences, $|r_k(\omega)| = |\sin(N\omega/2)/(N \sin(\omega/2))|$ [36].

B. Problem Formulation

For the k th BS, denote τ_k and ω_k as the group timing and frequency offsets respectively, and denote $\alpha_k^{(p)}$ denote the complex channel gain in the i th frame. Then, the received signal at time m can be expressed as

$$y[m] = \sum_{k=0}^{K-1} \sum_{p=0}^{P-1} \alpha_k^{(p)} c_k[m - pN_f - \tau_k] e^{-j\omega_k m} + \nu[m] \quad (4)$$

for all $m \in \{0, 1, \dots, N_o - 1\}$, where N_f is the frame length and $\nu[m] \sim \mathcal{CN}(0, \sigma_n^2)$ denotes additive white Gaussian noise.

¹The PSS and SSS alone suffice to identify the PCI and SSB index and to estimate the CFO and channel gain, as detailed in Section V.

The parameter N_o represents the total observation length under consideration. Without loss of generality, we assume that the received signal length is sufficiently large to accommodate all reference signals, and that synchronization sequences from different SSB do not overlap. Moreover, $c_k[m] = 0$ for $m \notin \{0, \dots, N-1\}$.

For notational convenience, we collect the received samples into a vector

$$\mathbf{y} \triangleq [y[0], y[1], \dots, y[N_o - 1]]^T \in \mathbb{C}^{N_o \times 1}. \quad (5)$$

The timing offsets and roots will be estimated and detected in pre-processing as detailed in Section V; in this part, the focus is on estimating the residual CFOs $\boldsymbol{\omega} \triangleq [\omega_k]_{k \in \mathcal{K}}$, the per-BS scaling factors $\boldsymbol{\mu} \triangleq [\mu_k]_{k \in \mathcal{K}}$, and the per-SSB channel gains $\boldsymbol{\alpha} \triangleq [\alpha_k^{(p)}]_{k \in \mathcal{K}, p \in \mathcal{P}}$. Formally, these parameters are obtained by solving the likelihood maximization problem

$$\mathcal{P}1 : \underset{\boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\mu}}{\text{maximize}} \quad \mathbb{P}(\mathbf{y} | \boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\mu}).$$

C. Multi-SSB Processing for Channel Estimation

For each SSB of the received signal, denoted as $\mathbf{y}^{(p)} \triangleq y[m + pN_f]_{m \in \{0, 1, \dots, N_f - 1\}} \in \mathbb{C}^{N_f \times 1}$, where $p \in \mathcal{P}$, then the p th received signal can be written as

$$\mathbf{y}^{(p)} = \underbrace{e^{-j\omega(p-1)N_f} \boldsymbol{\Omega}(\boldsymbol{\omega}) \odot \mathbf{C}(\boldsymbol{\tau})}_{\mathbf{A}^{(p)}} \boldsymbol{\alpha}^{(p)} + \mathbf{v}^{(p)}$$

where $\odot$ denotes the element-wise (Hadamard) product between vectors (or matrices) of compatible dimensions, $\mathbf{C}(\boldsymbol{\tau}) = [\mathbf{0}_{\tau_k \times 1}; \mathbf{c}_k; \mathbf{0}]_{k \in \mathcal{K}} \in \mathbb{C}^{N_f \times K}$, $\boldsymbol{\Omega}(\boldsymbol{\omega}) = [\Omega_k(\omega_k)]_{k \in \mathcal{K}} \in \mathbb{C}^{N_f \times K}$, $\Omega_k(\omega_k) = [1; e^{-j\omega_k}; \dots; e^{-j\omega_k(N_f-1)}]$, $\boldsymbol{\alpha}^{(p)} = [\alpha_k^{(p)}]_{k \in \mathcal{K}} \in \mathbb{C}^{K \times 1}$, and $\mathbf{v}^{(p)} = [v[m]]_{m \in \{0, \dots, N_f - 1\}} \in \mathbb{C}^{N_f \times 1}$.

As a result, the received signals $\mathbf{y}$ defined in (5) can be expressed as

$$\mathbf{y} = \underbrace{\text{blkdiag}(\mathbf{A}^{(0)}, \dots, \mathbf{A}^{(P-1)})}_{\mathbf{A}(\boldsymbol{\tau}, \boldsymbol{\omega})} \boldsymbol{\alpha} + \mathbf{v}. \quad (6)$$

where $\text{blkdiag}(\cdot)$ denotes the block-diagonal operator that arranges its matrix arguments along the main diagonal and fills all off-diagonal blocks with zeros.

Then, the problem $\mathcal{P}1$ can be equivalently transformed to a least-square estimation problem, and the channel gain $\boldsymbol{\alpha}$ can be estimated by

$$\hat{\boldsymbol{\alpha}} = \mathbf{A}(\boldsymbol{\tau}, \boldsymbol{\omega})^\dagger \mathbf{y} \quad (7)$$

where $\mathbf{A}^\dagger$ denotes the Moore-Penrose pseudoinverse of $\mathbf{A}$.

Similarly, the received signals $\mathbf{y}$ can be expressed as $\mathbf{A}_0 \boldsymbol{\alpha} + \underbrace{[\mathbf{A}^{(0)} \text{diag}(\boldsymbol{\alpha}^{(0)}); \dots; \mathbf{A}^{(P-1)} \text{diag}(\boldsymbol{\alpha}^{(P-1)})]}_{\mathbf{B}} \boldsymbol{\mu} + \mathbf{v}$.

where $\text{diag}(\cdot)$ denotes the diagonalization operator, $\mathbf{A}_i = \text{blkdiag}(\mathbf{A}_i^{(0)}, \dots, \mathbf{A}_i^{(P-1)})$, $\mathbf{A}_i^{(p)} = e^{-j\omega(p-1)N_f} \boldsymbol{\Omega}(\boldsymbol{\omega}) \odot \mathbf{C}_i(\boldsymbol{\tau})$, $\mathbf{C}_i(\boldsymbol{\tau}) = [\mathbf{0}_{(\tau+\tau_c i) \times 1}; \mathbf{c}_{k,i}; \mathbf{0}]_{k \in \mathcal{K}}$, and $\tau_c = N + \tau_0$. Then, according to the maximum likelihood (ML) estimation, the scaling factor $\boldsymbol{\mu}$ can be estimated by

$$\hat{\boldsymbol{\mu}} = \mathbf{B}^\dagger (\mathbf{y} - \mathbf{A}_0). \quad (8)$$

D. Multi-Burst Processing for CFO Estimation

We first characterize the correlation between the received signal and the reference signals. For an arbitrary SSB p of the received signal, we define the normalized cross-correlation between the received samples $\mathbf{y}$ and the two synchronization sequences $\mathbf{c}_{k,0}$ and $\mathbf{c}_{k,1}$ as

$$r_{y,k}^{p,i} \triangleq \frac{1}{N} \sum_{m=\tau}^{\tau+N-1} y[m + pN_f + i\tau_c] c_{k,i}^H[m - \tau_k] \quad (9)$$

where $k \in \mathcal{K}$, $p \in \mathcal{P}$, and $i \in \{0,1\}$.

Based on the cross-relation of the synchronization sequences, the distribution of $r_{y,k}^{p,i}$ is summarized in the following lemma.

Lemma 2. For any $k \in \mathcal{K}$, $p \in \mathcal{P}$, and $i \in \{0,1\}$, the correlation output $r_{y,k}^{p,i}$ follows

$$r_{y,k}^{p,i} \sim \alpha_k^{(p)} r_k(\omega_k) (\mu_k e^{-j\omega_k \tau_c})^i + \mathcal{CN}(0, \sigma_{k,p,i}^2) \quad (10)$$

where the variance $\sigma_{k,p,i}^2$ collects the averaged interference-plus-noise power

$$\sigma_{k,p,i}^2 = \frac{\sum_{q \neq k, q \in \mathcal{K}} \left(\mu_k^i \alpha_q^{(p)} \sigma_c \right)^2 + \sigma_n^2}{N}.$$

Proof. See Appendix B. $\square$

Lemma 2 shows that each correlation output $r_{y,k}^{p,i}$ can be interpreted as a noisy observation of a common complex coefficient $\alpha_k^{(p)} r_k(\omega_k)$, modulated by the exponential term $(\mu_k e^{-j\omega_k \tau_c})^i$ associated with the CFO ω_k . In particular, for a fixed (k, p) , the pair $(r_{y,k}^{p,0}, r_{y,k}^{p,1})$ shares the same unknown complex gain $\alpha_k^{(p)} r_k(\omega_k)$ while the CFO only appears through a deterministic phase rotation between the two correlations. This structure implies that the ratio between the two correlation outputs suppresses the unknown amplitude and reveals the CFO-dependent phase term.

More precisely, taking expectations in (10) yields $\mathbb{E}[r_{y,k}^{p,1}] / (\mu_k \mathbb{E}[r_{y,k}^{p,0}]) = e^{-j\omega_k \tau_c}$ and the phase of this ratio uniquely determines the CFO. This observation leads to the following single-SSB CFO estimator

$$\hat{\omega}_k^{\text{single}} = -\frac{1}{\tau_c} \angle \left(\frac{\mathbb{E}[r_{y,k}^{p,1}]}{\mu_k \mathbb{E}[r_{y,k}^{p,0}]} \right) = -\frac{1}{\tau_c} \angle \left(\frac{\mathbb{E}[r_{y,k}^{p,1}]}{\mathbb{E}[r_{y,k}^{p,0}]} \right) \quad (11)$$

the second equation holds because μ_k is real.

To exploit all SSBs within the CFO-coherent block jointly, we collect the correlation outputs associated with BS k into the vector $\mathbf{r}_{y,k} = [r_{y,k}^{p,i}]_{p \in \mathcal{P}, i \in \{0,1\}}$, and define the multi-SSB likelihood $\mathbb{P}(\mathbf{r}_{y,k} | \omega_k)$. Since synchronization sequences from different SSBs are non-overlapping in time, the correlation outputs are conditionally independent given ω_k , which yields

$$\mathbb{P}(\mathbf{r}_{y,k} | \omega_k) = \prod_{p \in \mathcal{P}} \prod_{i \in \{0,1\}} \mathbb{P}(r_{y,k}^{p,i} | \omega_k).$$

According to Lemma 2, each $r_{y,k}^{p,i}$ is complex Gaussian, so the log-likelihood function reduces (up to an additive constant) to

$$\ln(\mathbb{P}(\mathbf{r}_{y,k} | \omega_k)) \propto - \sum_{p \in \mathcal{P}} \sum_{i \in \{0,1\}} \frac{|r_{y,k}^{p,i} - \alpha_k^{(p)} r_k(\omega_k) (\mu_k e^{-j\omega_k \tau_c})^i|^2}{\sigma_{k,p,i}^2}. \quad (12)$$

Maximizing this log-likelihood with respect to ω_k leads to a closed-form multi-SSB ML CFO estimator, which is stated in the following proposition.

Proposition 1. The multi-SSB ML CFO estimator is

$$\hat{\omega}_k^{\text{multi}} = \frac{1}{\tau_c} \angle \{ \psi_1 \psi_0^H \} \quad (13)$$

where

$$\psi_0 \triangleq \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} (r_{y,k}^{p,0})^H}{\sigma_{k,p,0}^2}, \quad \psi_1 \triangleq \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} \mu_k (r_{y,k}^{p,1})^H}{\sigma_{k,p,1}^2}. \quad (14)$$

Proof. See Appendix C. $\square$

Proposition 1 shows that the estimator coherently combines all SSB within the CFO-coherence block through weighted sum of the correlation outputs (via ψ_0 and ψ_1), where the weight of each SSB is proportional to its effective SINR, $\alpha_k^{(p)} / \sigma_{k,p,i}^2$. Thus, high-SINR SSBs have a stronger impact on the final CFO estimate.

E. Performance Guarantee

Next, we characterize the estimation accuracy of the proposed multi-SSB estimator.

Proposition 2. The multi-SSB ML CFO estimator $\hat{\omega}_k^{\text{multi}}$ is asymptotically unbiased and satisfies

$$\hat{\omega}_k^{\text{multi}} \stackrel{a}{\sim} \mathcal{N}(\omega_k, [\mathbf{I}^{-1}]_{11})$$

where $[\mathbf{I}^{-1}]_{11}$ is the Cramer-Rao lower bound (CRLB) for ω_k and is given by

$$[\mathbf{I}^{-1}]_{1,1} = \frac{\Gamma_0^{-1} + \Gamma_1^{-1}}{2\tau_c^2 |r_k(\omega_k)|^2} \quad (15)$$

with the aggregated SINR of the two synchronization sequences defined as

$$\Gamma_0 = \sum_{p \in \mathcal{P}} \frac{|\alpha_k^{(p)}|^2}{\sigma_{k,p,0}^2}, \quad \Gamma_1 = \sum_{p \in \mathcal{P}} \frac{|\mu_k \alpha_k^{(p)}|^2}{\sigma_{k,p,1}^2}. \quad (16)$$

Proof. See Appendix D. $\square$

Proposition 2 shows that the CRLB degrades with increasing interference (through $\sigma_{k,p,i}^2$) and with decreasing $|r_k(\omega_k)|^2$ (typically when $|\omega_k|$ is large), so a coarse-to-fine refinement strategy, that consists of initial estimation, pre-compensation and successive interference cancellation, followed by re-estimation, can progressively improve the effective SINR and reduce the residual offset, thereby lowers the achievable error bound.

Moreover, the variance of any unbiased CFO estimator is lower-bounded by a term that is inversely proportional to the aggregated SINR across all SSBs. The quantities Γ_0 and Γ_1 collect, for each synchronization sequence, the per-SSB received SINR $|\alpha_k^{(p)}|^2/\sigma_{k,p,i}^2$. As a result, SSBs experiencing strong multi-cell interference (large $\sigma_{k,p,i}^2$) contribute only marginally to Γ_i , whereas high-SINR SSBs dominate the information on ω_k . This implies that accurate CFO estimation for a weak beam is still possible as long as at least one SSB within the CFO-coherence block has sufficiently high SINR, since the error variance is governed by the aggregate terms Γ_i , rather than by the weakest individual one. This dependence becomes explicit under a same-SINR scenario, as stated in the following theorem.

Theorem 2. *If all SSBs of BS k experience the same SINR, so that $|\alpha_k^{(p)}|^2/\sigma_{k,p,i}^2 = \rho_i$ for all $p \in \mathcal{P}$, then the aggregated SINR $\Gamma_i = P\rho_i$ and the CRLB for ω_k becomes*

$$[\mathbf{I}^{-1}]_{1,1} = \frac{\rho_0^{-1} + \rho_1^{-1}}{2P\tau_c^2 |r_k(\omega_k)|^2} \propto \frac{1}{P} \quad (17)$$

so that the minimum achievable variance decreases proportionally to $1/P$ as the number of SSBs P in the CFO-coherence block increases.

Thus, as more beams become available in future systems and more spatial directions are probed within each CFO-coherence block, the proposed estimator can exploit a larger effective P and achieve proportionally better CFO estimation accuracy. In addition, multiple SS burst sets can be jointly processed to further increase the total number of usable SSBs, if they are in CFO-coherence block. To make this dependence explicit, we next relate the estimation variance to the sampling parameters of the measurement platform.

Taking the motion model in Fig. 5, the effective number of SSBs and the corresponding drift-limited CRLB are given in the following theorem.

Theorem 3. *Denote $\delta_{\max}$ as the coherence limit on the CFO drift, the effective number of SSBs that can be coherently exploited is*

$$P_{\text{eff}} \approx \frac{\delta_{\max} \lambda_c d}{v^2 \sin^2 \theta \tau_p} P$$

where τ_p is the SS-burst period. Under the same-SINR assumption as Theorem 2, the corresponding drift-limited CRLB for ω_k satisfies

$$[\mathbf{I}^{-1}]_{1,1} \approx \frac{\tau_p (\rho_0^{-1} + \rho_1^{-1})}{2P\lambda_c \delta_{\max} \tau_c^2 |r_k(\omega_k)|^2} \frac{v^2 \sin^2 \theta}{d} \propto \frac{v^2 \sin^2 \theta}{d}.$$

Theorem 3 shows that, for a fixed coherence threshold $\delta_{\max}$, the drift-limited CRLB scales as $v^2 \sin^2 \theta / d$. First, the $\sin^2 \theta$ term indicates that motion with a large tangential component to the LOS (e.g., $\theta \approx \pi/2$) is the most detrimental, while radial motion ($\theta = 0$ or $\theta = \pi$) minimizes the drift-induced penalty. Hence, CFO-sensitive sampling should preferably follow LOS “rays” rather than tangential paths. Second, the dependence on v^2/d implies that, to maintain a given estimation accuracy, the sampling platform should slow down when it is close to the transmitter (small d), and can move faster when it is farther

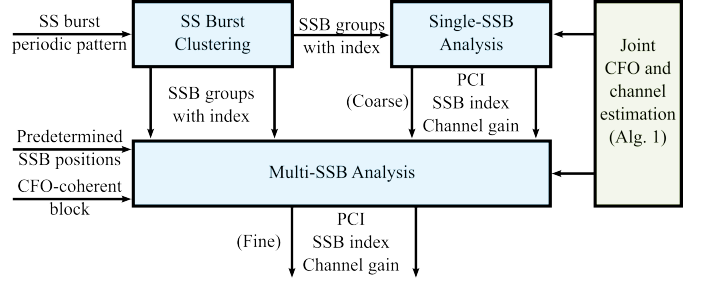


Figure 7. Proposed cross-burst detection and estimation framework. The algorithm follows a three-stage coarse-to-fine strategy, including SS-burst clustering, single-burst SIC-based analysis, and cross-burst refinement, that exploits predetermined SSB positions and CFO coherence to recover weak beams and produce refined PCI, SSB-index, and channel estimates.

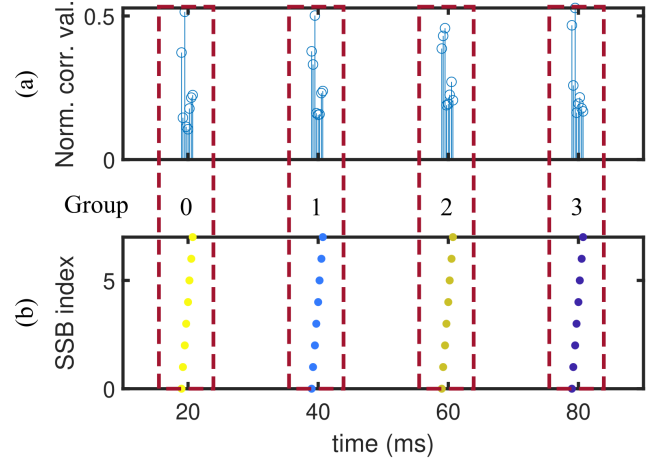


Figure 8. SSB index identification via temporal clustering of SS bursts. The process consists of three stages: 1) identifying candidate timing markers from the PSS correlation envelope (a); 2) grouping these markers by their periodic pattern into distinct clusters (colors in b); and 3) assigning an SSB index to each marker based on its relative timing within its assigned cluster (vertical position in b).

away and closer to radial motion. This suggests a distance-aware speed control to balance sampling efficiency and CFO accuracy. Finally, in the idealized limit ($v \rightarrow 0$) or ($\theta \in \{0, \pi\}$) and in the absence of other drift sources, the coherence block (and thus P_{eff}) becomes unbounded and the CRLB tends to zero.

V. IMPLEMENTATION

We propose a cross-burst detection and estimation framework, illustrated in Fig. 7, that exploits the structured nature of the SS burst set. On the detection side, a non-cascaded procedure is proposed that jointly processes multiple SSBs to improve detection efficiency and robustness in multi-cell interference. On the estimation side, we leverage CFO coherence within the burst to construct multi-SSB estimators, thereby enhancing the accuracy of CFO and channel-gain estimation, especially for weak, interference-masked beams.

A. SSB Identification via Timing Information

We propose an SSB-index identification method that exploits the temporal regularity of the SS burst set and

does not rely on prior PCI detection or highly accurate timing/frequency-offset estimates. The key observation is that inter-BS timing offsets are much smaller than the spacing between SSBs, since neighboring BSs are quasi-synchronous and the propagation-delay differences over typical inter-site distances are negligible at the SSB time scale. Consequently, the signal received in the i th intra-SSB time window is dominated by the superposition of the i th SSBs from nearby BSs, so each segment can be indexed by its position within the burst pattern, independent of the PCI, as shown in Fig. 4(a). In addition, the inter-burst period (e.g., 20 ms) is much longer than the intra-burst spacing (within 2.5 ms), so standard time-domain clustering can be used to group SSBs by burst and then assign distinct intra-burst indices.

To extract representative timing markers for each SSB, we first compute the PSS-based correlation envelope

$$\Lambda(t) \triangleq \max_{l \in \{0,1,2\}} \left| \frac{1}{N} \sum_{m=0}^{N-1} y[t+m] \left(c_{\text{PSS}}^{(l)}[t] \right)^* \right|^2$$

where $c_{\text{PSS}}^{(l)}[t]$ presents the l th PSS time-domain sequence. A structure-aware non-maximum suppression is then applied: a local maximum t_x is retained only if it is the unique dominant peak within a guard window that is long enough to cover relative delays across BSs, yet short enough to exclude the PSS of adjacent SSBs. The surviving peaks serve as timing markers, which are clustered according to the known SS-burst periodicity and then mapped to their relative intra-SSB positions, yielding SSB-index assignments, as shown in Fig. 8, that are PCI-agnostic and robust to residual timing and frequency offsets.

B. Complete Timing Offset Estimation and Full SSB Detection

Denote the set of estimated SSB indices and timing offsets via single-SSB analysis for group g and BS k by

$$\hat{\tau}_{g,k}^{\text{single}} \triangleq \left\{ \hat{\tau}_{g,k,p}^{\text{single}} \right\}_{p \in \mathcal{P}_{g,k}}$$

where $\hat{\tau}_{g,k,p}^{\text{single}}$ is the timing estimate for SSB p , and $\mathcal{P}_{g,k} \subseteq \mathcal{P}$ denotes the index set of detected SSBs in group g for the k th BS. Then, each detected and estimated delay $\hat{\tau}_{g,k,p}^{\text{single}}$ is able to yield a candidate estimate of the absolute timing of SSB 0 via

$$\hat{\tau}_0[p] = \hat{\tau}_{g,k,p}^{\text{single}} - \eta_p, \forall p \in \mathcal{P}_{g,k}.$$

Here, $\boldsymbol{\eta} = \{\eta_p\}_{p \in \mathcal{P}}$ with $\eta_0 = 0$ is the protocol-defined intra-SSB timing pattern.

Stacking all candidates, the absolute timing of SSB 0 can be obtained by the least-squares estimate

$$\begin{aligned} \hat{\tau}_0^{\text{cross}} &= \arg \min_x \frac{1}{|\mathcal{P}_{g,k}|} \sum_{p \in \mathcal{P}_{g,k}} |x - \hat{\tau}_0[p]|^2 \\ &= \frac{1}{|\mathcal{P}_{g,k}|} \sum_{p \in \mathcal{P}_{g,k}} \hat{\tau}_0[p]. \end{aligned}$$

The resulting estimates for all SSB positions in the (g, k) burst are then given by the shifted protocol grid

$$\hat{\tau}_{g,k,p}^{\text{cross}} = \hat{\tau}_0^{\text{cross}} + \eta_p, \forall p \in \mathcal{P}. \quad (18)$$

Algorithm 1 Joint CFO and Channel Estimation Algorithm

Initialization: $\hat{\tau} \leftarrow \hat{\tau}_{g,k,p}^{\text{cross}}, \hat{\omega} \leftarrow \mathbf{0}, \hat{\alpha} \leftarrow \mathbf{0}, \hat{\mu} \leftarrow \mathbf{1}, \tilde{\mathbf{y}} \leftarrow \mathbf{y}$

- 1) Channel Estimation: Construct $\mathbf{A}$, $\mathbf{A}_0$, and $\mathbf{B}$ as (7) and (8) based on $\hat{\tau}$, $\hat{\omega}$, $\hat{\alpha}$ and $\hat{\mu}$, then update channel parameters $\hat{\alpha}$ and $\hat{\mu}$ based on multi-SSB estimator (7) and (8).
- 2) SIC-based CFO Estimation: For each BS k , perform SIC and compensation: $\tilde{\mathbf{y}}_k \leftarrow (\mathbf{y} - \mathbf{A}(\hat{\tau}_{-k}, \hat{\omega}_{-k})\hat{\alpha}_{-k}) \odot \boldsymbol{\Omega}_k(\hat{\omega}_k)$. Then, estimate the remaining CFO δ_k based on the multi-SSB estimator (13), and update $\hat{\omega}_k \leftarrow \hat{\omega}_k + \delta_k$.
- 3) Go to step 1) until $\sum_k |\delta_k| < \epsilon$.

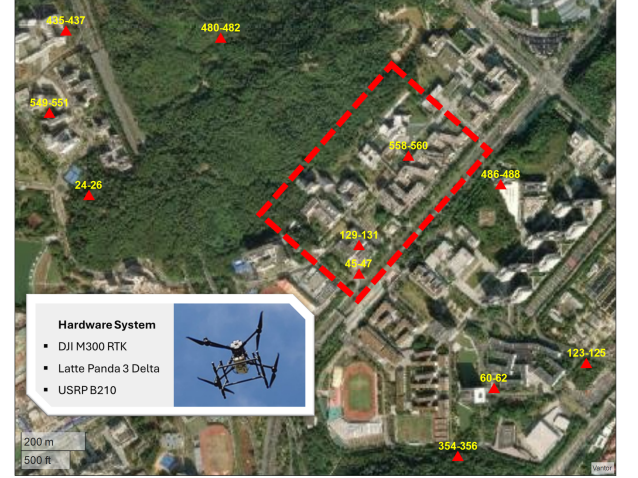


Figure 9. Aerial sampling campaign over the CUHK-Shenzhen campus. A DJI M300 RTK UAV equipped with an on-board Latte Panda 3 Delta computer, and a USRP B210 (calibrated) software-defined radio flies at an altitude of 150 m to collect measurements within the area outlined by the dashed red polygon. Red triangles with yellow labels indicate the locations and PCIs of a subset of known terrestrial BSs.

C. Joint CFO-Channel Estimation

The joint CFO and channel estimation procedure is summarized in Algorithm 1 and consists of two main steps. First, per-SSB channel gains and per-BS scaling factors are estimated from (7) and (8), then the CFO is obtained from (13), with these updates iterated until the residual CFO becomes negligible. This algorithm directly applies to the single-SSB case by setting $P = 1$.

To effectively refine the CFO estimates, an iterative SIC-based update is employed. For every $k \in \mathcal{K}$, the contributions of all other BSs are first canceled, the current CFO estimate of BS k is used for pre-compensation, and a residual CFO increment δ_k is then estimated from the cleaned waveform. The residual signal associated with BS k is

$$\tilde{\mathbf{y}}_k = \left(\mathbf{y} - \mathbf{A}(\hat{\tau}_{-k}, \hat{\omega}_{-k})\hat{\alpha}_{-k} \right) \odot \boldsymbol{\Omega}_k(\hat{\omega}_k) \quad (19)$$

where “ $-k$ ” denotes the collection of parameters for all BSs except k (i.e., the entries associated with BS k are set to zero) and $\boldsymbol{\Omega}_k(\hat{\omega}_k)$ is the CFO pre-compensation vector for BS k . The residual CFO is then updated as

$$\hat{\omega}_k \leftarrow \hat{\omega}_k + \delta_k$$

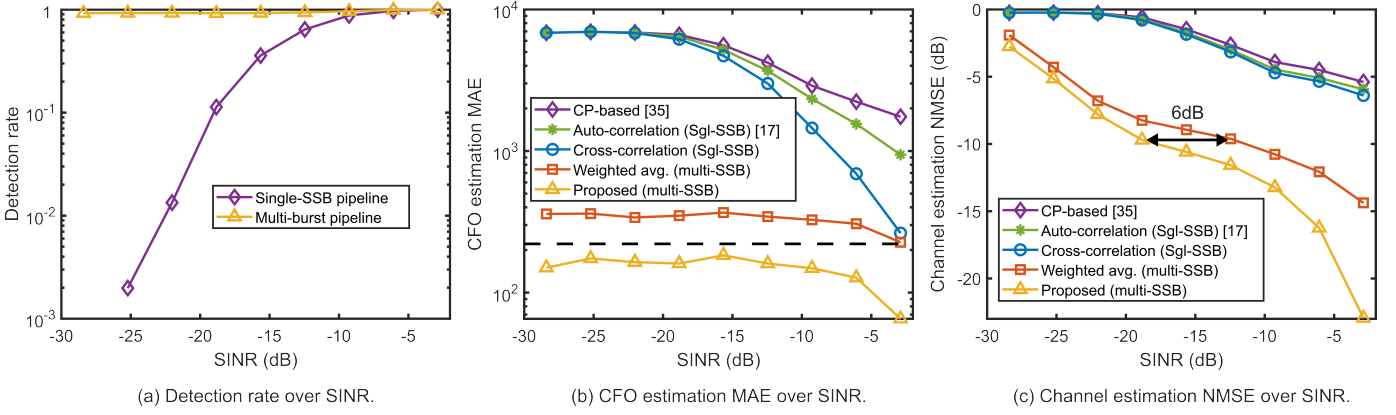


Figure 10. The detection rate, CFO estimation MAE, and Channel estimation NMSE over SINR.

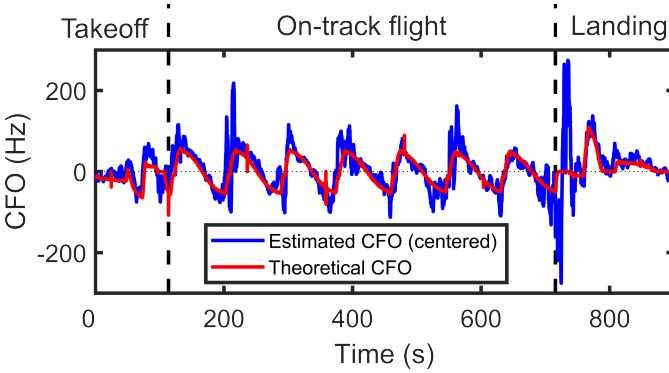


Figure 11. CFO fluctuation over time during the low-altitude radio-map measurements, illustrating the correlation between the estimated CFO (centered) and the theoretical CFO across the three flight phases: takeoff, on-track flight, and landing.

and the iteration terminates when the residual CFO corrections $\{\delta_k\}$ fall below a prescribed threshold.

Improved CFO estimates sharpen the effective matched SSBs and reduce the bias and variance of the channel estimates; the refined channel estimates and CFO yield smaller CFO increments by smaller $\sigma_{k,p,i}^2$ and larger $|r_k(\delta_k)|^2$. Under Proposition 2, this alternating update leads to a monotone decrease of the residual CFO.

VI. PERFORMANCE EVALUATION

In this section, both simulation and experimental platforms are developed to validate the proposed cross-burst detection and estimation framework. The simulation platform generates 5G NR waveforms based on a configurable system model [14]–[16] and is used to evaluate the robustness of the algorithm under various controlled scenarios. In contrast, the experimental platform collects real-world 5G waveform in field deployments, enabling assessment of the algorithm's effectiveness in practical systems for radio map construction.

A. Platform Settings

1) *Simulation platform:* The simulation platform generates standard-compliant 5G NR downlink waveforms. The

frequency-domain sequences for the PSS, and SSS are generated according to the 3GPP specifications detailed in Section II-A, and the composite baseband signal is synthesized via OFDM modulation. To emulate a low-altitude interference-limited scenario, we consider a configurable number of interfering BSs. For each BS k , the propagation delay $\tau_k \sim \mathcal{U}[0, \tau_{\max}]$ and the CFO $\omega_k \sim \mathcal{U}[-\omega_{\max}, \omega_{\max}]$ model asynchronous network deployment and oscillator offsets, while the per-SSB complex gain $\alpha_k^{(p)} \sim \mathcal{CN}(0, 1)$ follows a Rayleigh fading model. The received signal is the superposition of all SSBs from all BSs plus complex white Gaussian noise $v \sim \mathcal{CN}(0, \delta_n^2)$. Unless stated otherwise, each performance point is averaged over 200 Monte Carlo trials with $K = 12$ BSs and $P = 12$ SSBs.

2) *Experimental Platform:* A DJI M300 RTK UAV carries a universal software radio peripheral (USRP) to collect 5G NR signals, as illustrated in Fig. 9, with carrier frequency $f_c = 2.52495$ GHz and bandwidth 20 MHz.

B. Simulation Results

The detection performance of the proposed cross-burst framework is compared with the classical single-SSB baseline (as detailed in Section II-B) in Fig. 10(a). For both methods, the detection rate increases with the SINR, but the proposed framework consistently outperforms the baseline over the entire SINR range, with a significant gain in the low-SINR regime. This robustness gain is attributed to the exploitation of the cross-burst structure, where we use the strong SSBs as anchors to detect the weak SSBs, thereby leading to robust detection.

The estimation performance of the proposed framework is compared with four baselines: i) a CP-based blind estimator [35]; ii) an auto-correlation estimator that splits each synchronization sequence into two segments and obtains the CFO from their phase difference [17]; iii) the single-SSB cross-correlation estimator in (11); and iv) a multi-SSB baseline that forms a power-weighted average of the separately estimated CFOs.

Fig. 10(b) shows the mean absolute error (MAE) of the CFO estimate versus SINR. As SINR increases, the error decreases for all methods, while the proposed multi-SSB

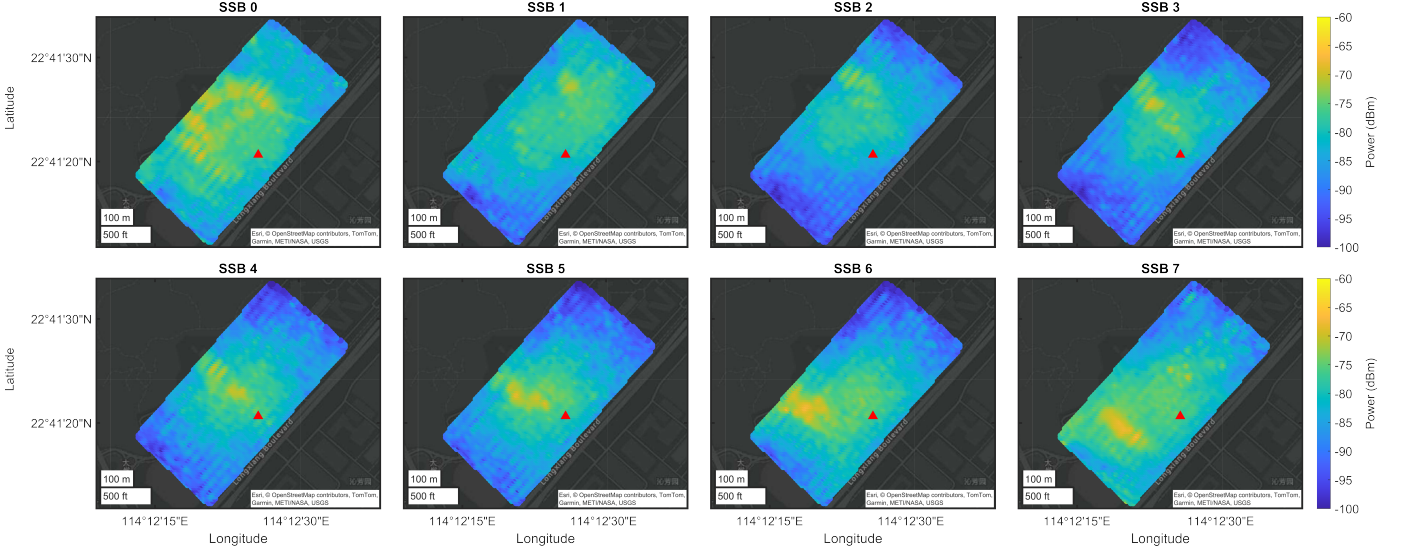


Figure 12. Per-beam low-altitude radio maps for all beams 0-7 of the cell with PCI 45, where the red rectangle marks the base-station location.

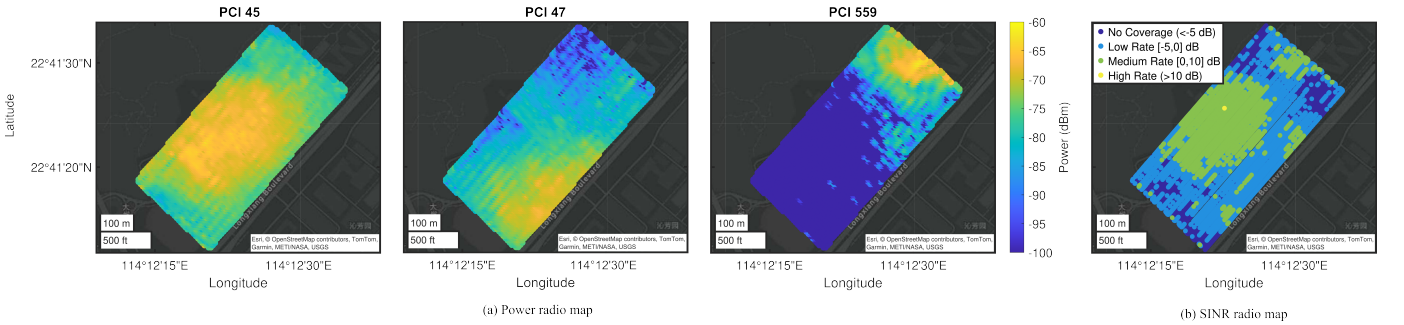


Figure 13. Per-cell low-altitude radio maps. (a) Power radio maps for representative cells. (b) Integrated SINR radio map combining all detectable cells.

estimation algorithms significantly outperforms the baselines. Remarkably, the proposed method at an SINR of -30 dB still exceeds the performance of the power-weighted baseline at -5 dB, which highlights its strong robustness in low-SINR conditions. Even in the single-SSB case, the proposed cross-correlation estimator outperforms auto-correlation because it exploits a longer phase baseline.

The improved CFO estimation directly benefits subsequent channel estimation. As illustrated in Fig. 10(c), the normalized mean squared error (NMSE) of the channel estimate exhibits the same ordering as the CFO MAE. This confirms that the more accurate CFO compensation provided by the proposed framework is crucial for high-fidelity channel characterization.

C. Field Test Results

We first validate the proposed estimator and the overall measurement system by comparing the Doppler-induced theoretical CFO drift with the demeaned CFO produced by our algorithm. According to the Doppler relation, the motion-induced CFO offset satisfies

$$\Delta f = -\frac{v_{\text{LOS}} f_c}{c}$$

where v_{LOS} is the projection of the UAV velocity onto the LOS direction. As shown in Fig. 11, during the on-track flight

phase, when the UAV follows the predetermined trajectory for radio-map sampling, the theoretical CFO and the estimated CFO (centered) closely match. The larger discrepancies around some peaks and valleys are mainly observed during turning maneuvers, where the received signal becomes less stable. Overall, the strong agreement between the two curves confirms that the proposed cross-burst framework can accurately track the CFO from 5G NR SS burst sets and validates the effectiveness of the entire measurement system.

Fig. 12 illustrates the per-beam radio maps in the 150 m low-altitude airspace. The coverage patterns of the eight beams can be clearly distinguished: beam 0 provides broad-area coverage, while beams 1 to 7 form seven narrower beams that sweep from approximately north toward west. The received power does not exhibit a simple monotonic distance- or propagation-loss trend but instead shows irregular spatial variations, since the dominant energy in the low-altitude airspace is largely contributed by ground-reflected paths. For the same reason, different beams exhibit different power levels in the airspace, which mainly reflect variations in the underlying ground environment; for example, beams 1-3 appear weaker in regions with fewer reflective structures, whereas beams 4-7 are stronger where more reflectors are present. These observations highlight the intrinsic complexity of low-altitude coverage by

terrestrial deployments.

Fig. 13 illustrates the per-cell radio maps in the 150 m low-altitude airspace. In Fig. 13(a), the received-power map for representative cells is shown, where at each position the maximum power over all beams of that cell is taken. The dominant coverage regions of different cells can be clearly distinguished: the cell with PCI 45 mainly serves the north-west sector, the cell with PCI 47 covers the southern sector, and the cell with PCI 559 primarily illuminates the eastern side. To assess low-altitude communication performance, Fig. 13(b) presents the corresponding SINR radio map, partitioned into four ranges, $\text{SINR} < -5$ dB, $[-5, 0]$ dB, $[0, 10]$ dB, and > 10 dB, which can be interpreted as no coverage, low-rate, medium-rate, and high-rate regions, respectively, in line with typical 3GPP link-budget guidelines [37]. The results show that ground base stations can provide wide-area low-rate coverage in the airspace via reflections, and medium-rate coverage around the main reflection regions. However, coverage holes appear in inter-cell areas where the reflected power is low and inter-cell interference is strong. Moreover, due to the large reflection loss and severe inter-cell interference (for example, between cells 45 and 47, whose ground beams are oriented towards different areas but whose reflected components overlap aloft) only very limited regions achieve high SINR. This observation highlights the inherent difficulty of sustaining high-rate links in interference-limited low-altitude deployments and underscores the need for coordinated terrestrial BS deployment that explicitly accounts for both ground and low-altitude users.

VII. CONCLUSION

This work proposed a successive waveform reconstruction and cancellation framework for constructing per-cell per-beam low-altitude radio maps from 5G NR SS burst sets. To support effective reconstruction and cancellation, we introduced the CFO-coherent block and developed a joint CFO-channel estimator that coherently aggregates multiple SSBs during the CFO-coherent block, together with closed-form accuracy scaling laws that relate estimation performance to UAV speed, motion geometry, burst periodicity, and the length of the CFO-coherent block. Simulations showed that the proposed method can detect and estimate synchronization signals at SINR levels down to -30 dB. Field measurements with a UAV-mounted receiver confirmed that the proposed method can reliably obtain per-cell per-beam low-altitude radio maps and revealed that, despite strong received power, the measured SINR rarely exceeds 10 dB. These results provide a fundamental basis for the design of aerial communication systems and highlight the critical need for interference-aware design and optimization of terrestrial deployments to support future low-altitude aerial services.

APPENDIX A POOF OF LEMMA 1

At $t = 0$, the receiver is located at $\mathbf{x}[0] = (d, 0)$ as shown in Fig. 5. We parameterize the velocity direction in the LOS-based spherical coordinates as $\mathbf{v} = [\cos \theta, \sin \theta]^T v$. Thus, at time τ , the receiver position is

$$\mathbf{x}[\tau] = \mathbf{x}[0] + \mathbf{v}\tau = (d + v\tau \cos \theta, v\tau \sin \theta).$$

Then the instantaneous angle between $\mathbf{v}[\tau]$ and the new LOS is

$$\theta(\tau) = \frac{\langle \mathbf{v}, \mathbf{x}[\tau] \rangle}{\|\mathbf{v}\| \|\mathbf{x}[\tau]\|} = \frac{d \cos \theta + v\tau}{\sqrt{d^2 + 2dv\tau \cos \theta + v^2\tau^2}}.$$

Under the classical Doppler model, the CFO drift over the interval τ is

$$\Delta f = f[\tau] - f[0] = \frac{v}{\lambda_c} (\cos \theta(\tau) - \cos \theta) \quad (20)$$

where λ_c is carrier wavelenth. Introduce $\epsilon \triangleq v\tau/d$, then $\theta(\tau)$ can be rewritten as

$$\theta(\tau) = \frac{\cos \theta + \epsilon}{\sqrt{1 + 2\epsilon \cos \theta + \epsilon^2}}.$$

When $\epsilon \rightarrow 0$, that is $v\tau \ll d$, $\theta(\tau)$ can be approximated, by first-order Taylor expansion around $\epsilon = 0$, as

$$\theta(\tau) \approx \cos \theta + \epsilon \sin^2 \theta \quad (21)$$

where the approximation error is of order $\mathcal{O}(v^2\tau^2/d^2)$.

Substituting (21) into (20), we have

$$\Delta f \approx \frac{v^2}{d} \frac{\tau}{\lambda_c} \sin^2 \theta. \quad (22)$$

APPENDIX B PROOF OF LEMMA 2

Start from the first synchronization sequence, that is, $i = 0$, according to the definition of $y[m]$ in (4), the cross-correlation can be expressed as

$$\begin{aligned} r_{y,k}^{p,0} &= \frac{1}{N} \sum_{m=\tau_k}^{\tau_k+N-1} \left(\sum_{q=1}^K \alpha_q^{(p)} c_{q,0} [m - \tau_q] e^{-j\omega_q m} + \nu[m] \right) \\ &\quad \times c_{k,0}^H [m - \tau_k] \end{aligned}$$

Decompose the sum into: i) desired signal $q = k$, ii) multi-BS interference $q \neq k$, and iii) noise.

For $q = k$, the cross-correlation for desired signal is

$$\begin{aligned} \frac{1}{N} \sum_{m=\tau_k}^{\tau_k+N-1} \alpha_k^{(p)} c_{k,0} [m - \tau_k] e^{-j\omega_k m} c_{k,0}^H [m - \tau_k] \\ = \alpha_k^{(p)} r_{k,k}(\tau_k, \omega_k) = \alpha_k^{(p)} r_k(\omega_k). \end{aligned}$$

For $q \neq k$, the cross-correlation for interference signal is

$$\begin{aligned} \sum_{q \neq k} \frac{1}{N} \sum_{m=\tau_k}^{\tau_k+N-1} \alpha_q^{(p)} c_{q,0} [m - \tau_q] e^{-j\omega_q m} c_{k,0}^H [m - \tau_k] \\ = \sum_{q \neq k} \alpha_q^{(p)} r_{k,q}(\tau_k - \tau_q, \omega_q) = \mathcal{CN} \left(0, \sum_{q \neq k} \alpha_q^{(p)2} \sigma_c^2 / N \right). \end{aligned}$$

For the noise, the cross-correlation is

$$\begin{aligned} \frac{1}{N} \sum_{m=\tau_k}^{\tau_k+N-1} \nu[m] c_{k,0}^H [m - \tau_k] \\ = \mathcal{CN} \left(0, \frac{1}{N} \sum_{m=0}^{N-1} |c_{k,0}[m]|^2 \sigma_n^2 \right) = \mathcal{CN}(0, \sigma_n^2 / N). \end{aligned}$$

Putting i) to iii) together gives

$$r_{y,k}^{p,0} = \alpha_k^{(p)} r_k(\omega_k) + \mathcal{CN}(0, \sigma_{k,p,0}^2)$$

where $\sigma_{k,p,0}^2 = (\sum_{q \neq k} |\alpha_q^{(p)} \sigma_c|^2 + \sigma_n^2)/N$.

Similarly, consider $i = 1$, we have

$$r_{y,k}^{p,1} = \mu_k \alpha_k^{(p)} r_k(\omega_k) e^{-j\omega_k \tau_c} + \mathcal{CN}(0, \sigma_{k,p,1}^2)$$

where $\sigma_{k,p,1}^2 = (\sum_{q \neq k} |\mu_k \alpha_q^{(p)} \sigma_c|^2 + \sigma_n^2)/N$.

In summary, the cross-correlation $r_{y,k}^{p,i}$ can be expressed as

$$r_{y,k}^{p,i} = \alpha_k^{(p)} r_k(\omega_k) (\mu_k e^{-j\omega_k \tau_c})^i + \mathcal{CN}(0, \sigma_{k,p,i}^2)$$

with $\sigma_{k,p,i}^2 = (\sum_{p \neq k} |\mu_k \alpha_q^{(p)} \sigma_c|^2 + \sigma_n^2)/N$.

APPENDIX C PROOF OF PROPOSITION 1

Since the autocorrelation function $r_k(\omega_k)$ depends on the CFO variable ω_k , we introduce an auxiliary variable $\gamma_k \triangleq r_k(\omega_k)$ to absorb the CFO dependence. Consequently, the unknown parameters now consist of $\theta \triangleq [\omega_k, \gamma_k, \gamma_k^H]^T$, where ω_k is real and (γ_k, γ_k^H) is complex. Using this reparametrization, the log-likelihood function (12) becomes

$$f(\theta) = \sum_{p \in \mathcal{P}} -\frac{|r_{y,k}^{p,0} - \alpha_k^{(p)} \gamma_k|^2}{\sigma_{k,p,0}^2} + \sum_{p \in \mathcal{P}} -\frac{|r_{y,k}^{p,1} - \alpha_k^{(p)} \gamma_k \mu_k e^{-j\omega_k \tau_c}|^2}{\sigma_{k,p,1}^2}. \quad (23)$$

Denote the cumulative average SINR for each reference signal as

$$\Gamma_0 = \sum_{p \in \mathcal{P}} \frac{|\alpha_k^{(p)}|^2}{\sigma_{k,p,0}^2}, \quad \Gamma_1 = \sum_{p \in \mathcal{P}} \frac{|\mu_k \alpha_k^{(p)}|^2}{\sigma_{k,p,1}^2} \quad (24)$$

and the cumulative SINR realization for each reference signal as

$$\psi_0 \triangleq \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} (r_{y,k}^{p,0})^H}{\sigma_{k,p,0}^2}, \quad \psi_1 \triangleq \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} \mu_k (r_{y,k}^{p,1})^H}{\sigma_{k,p,1}^2}. \quad (25)$$

Then, taking differentiation for $(\omega_k, \gamma_k, \gamma_k^H)$, we have

$$\frac{\partial f}{\partial \omega_k} = 2\tau_c \text{Im} \{ \psi_1 \gamma_k e^{-j\omega_k \tau_c} \} \quad (26)$$

$$\frac{\partial f}{\partial \gamma_k} = \psi_0 + \psi_1 e^{-j\omega_k \tau_c} - (\Gamma_0 + \Gamma_1) \gamma_k^H \quad (27)$$

$$\frac{\partial f}{\partial \gamma_k^H} = \psi_0^H + \psi_1^H e^{j\omega_k \tau_c} - (\Gamma_0 + \Gamma_1) \gamma_k. \quad (28)$$

Taking $\frac{\partial f}{\partial \omega_k} = 0$, we have

$$\omega_k \tau_c = \angle \{ \psi_1 \gamma_k \} \quad (29)$$

and taking $\frac{\partial f}{\partial \gamma_k^H} = 0$, we have

$$\gamma_k = \frac{\psi_0^H + \psi_1^H e^{j\omega_k \tau_c}}{(\Gamma_0 + \Gamma_1)}. \quad (30)$$

Substituting (30) into (29), we have

$$\omega_k \tau_c = \angle \left\{ \frac{\psi_1 \psi_0^H + |\psi_1^H|^2 e^{j\omega_k \tau_c}}{(\Gamma_0 + \Gamma_1)} \right\} \stackrel{(a)}{=} \angle \left\{ \frac{\psi_1 \psi_0^H}{|\psi_1^H|^2} + e^{j\omega_k \tau_c} \right\}$$

where (a) holds because $\angle \{ \alpha x \} = \angle \{ x \}$ for any non-zero real α and here we set $\alpha = (\Gamma_0 + \Gamma_1)/|\psi_1^H|^2$. Then, we have

$$\begin{aligned} & \angle \left\{ \frac{\psi_1 \psi_0^H}{|\psi_1^H|^2} + e^{j\omega_k \tau_c} \right\} - \omega_k \tau_c \\ &= \angle \left\{ \left(\frac{\psi_1 \psi_0^H}{|\psi_1^H|^2} e^{-j\omega_k \tau_c} + 1 \right) \right\} = 0 \end{aligned}$$

which means $\angle \left\{ \left(\frac{\psi_1 \psi_0^H}{|\psi_1^H|^2} e^{-j\omega_k \tau_c} \right) \right\} = 0$ then

$$\omega_k \tau_c = \angle \left\{ \frac{\psi_1 \psi_0^H}{|\psi_1^H|^2} \right\} = \angle \{ \psi_1 \psi_0^H \}.$$

As a result, the ML estimator is

$$\omega_k = \angle \{ \psi_1 \psi_0^H \} / \tau_c.$$

APPENDIX D PROOF OF PROPOSITION 2

According to the asymptotic property of the ML estimator theorem in [38], we have $\hat{\omega}_k^{\text{multi}} \stackrel{a}{\sim} \mathcal{N}(\omega_k, \text{CRLB})$. Taking the second-order of the log-likelihood function (12), we have the second derivative of f over θ is

$$\begin{bmatrix} -2\tau_c^2 \text{Re} \{ \psi_1 \gamma_k e^{-j\omega_k \tau_c} \} & -j\tau_c \psi_1 e^{-j\omega_k \tau_c} & j\tau_c \psi_1^H e^{j\omega_k \tau_c} \\ -j\tau_c \psi_1 e^{-j\omega_k \tau_c} & 0 & -(\Gamma_0 + \Gamma_1) \\ j\tau_c \psi_1^H e^{j\omega_k \tau_c} & -(\Gamma_0 + \Gamma_1) & 0 \end{bmatrix}.$$

Next, we calculate the exception of the $\frac{\partial^2 f}{\partial^2 \theta}$ to obtain the Fisher information matrix (FIM). First,

$$\begin{aligned} & \mathbb{E} \{ -2\tau_c^2 \text{Re} \{ \psi_1 \gamma_k e^{-j\omega_k \tau_c} \} \} \\ &= \mathbb{E} \left\{ -2\tau_c^2 \text{Re} \left\{ \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} \mu_k (r_{y,k}^{p,1})^H}{\sigma_{k,p,1}^2} \gamma_k e^{-j\omega_k \tau_c} \right\} \right\} \\ &= -2\tau_c^2 \text{Re} \left\{ \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} \mu_k \mathbb{E} \{ (r_{y,k}^{p,1})^H \}}{\sigma_{k,p,1}^2} \gamma_k e^{-j\omega_k \tau_c} \right\} \\ &= -2\tau_c^2 \text{Re} \left\{ \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} \mu_k (\alpha_k^{(p)} \gamma_k \mu_k e^{-j\omega_k \tau_c})^H}{\sigma_{k,p,1}^2} \gamma_k e^{-j\omega_k \tau_c} \right\} \\ &= -2\tau_c^2 \text{Re} \left\{ \sum_{p \in \mathcal{P}} \frac{|\alpha_k^{(p)} \mu_k|^2}{\sigma_{k,p,1}^2} |\gamma_k|^2 \right\} = -2\tau_c^2 \Gamma_1 |\gamma_k|^2 \end{aligned}$$

Similarly, we have $\mathbb{E} \{ -j\tau_c \psi_1 e^{-j\omega_k \tau_c} \} = -j\tau_c \Gamma_1 \gamma_k^H$ and $\mathbb{E} \{ j\tau_c \psi_1^H e^{j\omega_k \tau_c} \} = j\tau_c \Gamma_1 \gamma_k$.

Accordingly, the FIM of (23) is

$$\mathbf{I} = -\mathbb{E} \left\{ \frac{\partial^2 f}{\partial^2 \theta} \right\} = \begin{bmatrix} 2\tau_c^2 \Gamma_1 |\gamma_k|^2 & j\tau_c \Gamma_1 \gamma_k^H & -j\tau_c \Gamma_1 \gamma_k \\ j\tau_c \Gamma_1 \gamma_k^H & 0 & \Gamma_0 + \Gamma_1 \\ -j\tau_c \Gamma_1 \gamma_k & \Gamma_0 + \Gamma_1 & 0 \end{bmatrix}.$$

Denote $\mathbf{I} \triangleq [a, \mathbf{b}^H; \mathbf{b}, \mathbf{D}]$, with $a = 2\tau_c^2 \Gamma_1 |\gamma_k|^2$, $\mathbf{b} = [j\tau_c \Gamma_1 \gamma_k^H, -j\tau_c \Gamma_1 \gamma_k]^T$, and

$$\mathbf{D} = \begin{bmatrix} 0 & \Gamma_0 + \Gamma_1 \\ \Gamma_0 + \Gamma_1 & 0 \end{bmatrix}.$$

By applying inverse via Schur complement, the CRLB for CFO ω_k is

$$[\mathbf{I}^{-1}]_{11} = (a - \mathbf{b}^H \mathbf{D}^{-1} \mathbf{b})^{-1} = \frac{\Gamma_0^{-1} + \Gamma_1^{-1}}{2\tau_c^2 |\gamma_k|^2}.$$

Replace γ_k by $r_k(\omega_k)$, we have the CRLB as

$$[\mathbf{I}^{-1}]_{11} = \frac{\Gamma_0^{-1} + \Gamma_1^{-1}}{2\tau_c^2 |r_k(\omega_k)|^2}.$$

REFERENCES

- [1] M. Song, Y. Lin, J. Wang, G. Sun, C. Dong, N. Ma, D. Niyato, and P. Zhang, "Trustworthy intelligent networks for low-altitude economy," *IEEE Commun. Mag.*, vol. 63, no. 7, pp. 72–79, 2025.
- [2] Y. Jiang, X. Li, G. Zhu, H. Li, J. Deng, K. Han, C. Shen, Q. Shi, and R. Zhang, "Integrated sensing and communication for low altitude economy: Opportunities and challenges," *IEEE Commun. Mag.*, pp. 1–7, 2025.
- [3] J. Yang, M. Ding, G. Mao, Z. Lin, D.-G. Zhang, and T. H. Luan, "Optimal base station antenna downtilt in downlink cellular networks," *IEEE Trans. on Wireless Commun.*, vol. 18, no. 3, pp. 1779–1791, 2019.
- [4] M. U. C. Md, G. Ismail, S. Walid, and B. Arupjyoti, "Ensuring reliable connectivity to cellular-connected UAVs with up-tilted antennas and interference coordination," *ITU Journal on Future and Evolving Technologies*, vol. 2, no. 2, pp. 165–185, 2021.
- [5] G. Geraci, A. Garcia-Rodriguez, M. M. Azari, A. Lozano, M. Mezzavilla, S. Chatzinotas, Y. Chen, S. Rangan, and M. D. Renzo, "What will the future of UAV cellular communications be? a flight from 5G to 6G," *IEEE Commun. Surveys Tuts.*, vol. 24, no. 3, pp. 1304–1335, 2022.
- [6] J. Chen, B. Li, H. Sun, S. Cui, and N. Pappas, "Predictive communications for low-altitude networks," 2025. [Online]. Available: <https://arxiv.org/abs/2509.01705>
- [7] S. Chai and V. K. Lau, "Mixed-timescale request-driven user association, trajectory and radio resource control for cache-enabled multi-UAV networks," *IEEE Trans. Signal Process.*, vol. 70, pp. 4997–5011, 2022.
- [8] Z. Zhou, Y. Zeng, C. Li, F. Yang, Y. Chen, and J. Joung, "Full-dimensional beamforming for multi-user MIMO-OFDM ISAC for low-altitude UAV with zero sensing resource allocation," 2025. [Online]. Available: <https://arxiv.org/abs/2508.06428>
- [9] D. Romero and S.-J. Kim, "Radio map estimation: A data-driven approach to spectrum cartography," *IEEE Signal Process. Mag.*, vol. 39, no. 6, pp. 53–72, 2022.
- [10] H. Sun and J. Chen, "Integrated interpolation and block-term tensor decomposition for spectrum map construction," *IEEE Trans. Signal Process.*, vol. 72, pp. 3896–3911, 2024.
- [11] S. Shrestha, X. Fu, and M. Hong, "Deep spectrum cartography: Completing radio map tensors using learned neural models," *IEEE Trans. Signal Process.*, vol. 70, pp. 1170–1184, 2022.
- [12] H. Wang, J. Zhang, G. Nie, L. Yu, Z. Yuan, T. Li, J. Wang, and G. Liu, "Digital twin channel for 6G: Concepts, architectures and potential applications," *IEEE Commun. Mag.*, vol. 63, no. 3, pp. 24–30, 2025.
- [13] D. He, B. Ai, K. Guan, L. Wang, Z. Zhong, and T. Kürner, "The design and applications of high-performance ray-tracing simulation platform for 5G and beyond wireless communications: A tutorial," *IEEE Commun. Surveys Tuts.*, vol. 21, no. 1, pp. 10–27, 2019.
- [14] 3rd Generation Partnership Project (3GPP), "NR; Radio Resource Control (RRC); Protocol specification," 3GPP, Technical Specification (TS) 38.331, Jul 2025, release 18.
- [15] —, "NR; Physical layer procedures for control," 3GPP, Technical Specification (TS) 38.213, Jul 2025, release 18.
- [16] —, "NR; Physical channels and modulation," 3GPP, Technical Specification (TS) 38.211, Apr 2025, release 18.
- [17] R. Tuninato, D. G. Riviello, R. Garelli, B. Melis, and R. Fantini, "A comprehensive study on the synchronization procedure in 5G NR with 3GPP-compliant link-level simulator," *EURASIP J. Wireless Com. Network*, vol. 2023, no. 1, p. 111.
- [18] X. Zhou, L. Chen, Y. Ruan, and R. Chen, "Indoor localization with multi-beam of 5G new radio signals," *IEEE Trans. on Wireless Commun.*, vol. 23, no. 9, pp. 11 260–11 275, 2024.
- [19] S. Yoneda, M. Sawahashi, S. Nagata, and S. Suyama, "Prach transmission employing carrier frequency offset pre-compensation based on measurement at UE for NR uplink," *IEICE Trans. on Commun.*, vol. E108-B, no. 3, pp. 330–338, 2025.
- [20] W. Dong, L. Chen, Z. Ju, T. Zhou, Z. Liu, and R. Chen, "Beam-switching-based time-of-arrival ranging on commercial 5G NR signals for outdoor positioning," *IEEE Trans. on Wireless Commun.*, pp. 1–1, 2025.
- [21] MathWorks, "NR cell search and MIB and SIB1 recovery," <https://se.mathworks.com/help/5g/ug/nr-cell-search-and-mib-and-sib1-recovery.html>, 2025, accessed: 2025-11-03.
- [22] M. Morelli, C.-C. J. Kuo, and M.-O. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review," *Proc. IEEE*, vol. 95, no. 7, pp. 1394–1427, 2007.
- [23] J. Park and J. Chun, "Improved lattice reduction-aided MIMO successive interference cancellation under imperfect channel estimation," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 3346–3351, 2012.
- [24] J. van de Beek, M. Sandell, and P. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. Signal Process.*, vol. 45, no. 7, pp. 1800–1805, 1997.
- [25] T. Lv, H. Li, and J. Chen, "Joint estimation of symbol timing and carrier frequency offset of ofdm signals over fast time-varying multipath channels," *IEEE Trans. Signal Process.*, vol. 53, no. 12, pp. 4526–4535, 2005.
- [26] A. A. Nasir, H. Mehrpouyan, S. D. Blostein, S. Durrani, and R. A. Kennedy, "Timing and carrier synchronization with channel estimation in multi-relay cooperative networks," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 793–811, 2012.
- [27] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, 2010.
- [28] Y. Xu, E. G. Larsson, E. A. Jorswieck, X. Li, S. Jin, and T.-H. Chang, "Distributed signal processing for extremely large-scale antenna array systems: State-of-the-art and future directions," *IEEE Trans. Signal Process.*, vol. 19, no. 2, pp. 304–330, 2025.
- [29] B. W. Zarikoff and J. K. Cavers, "Multiple frequency offset estimation for the downlink of coordinated MIMO systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 6, pp. 901–912, 2008.
- [30] H. Wang, X.-G. Xia, and Q. Yin, "Computationally efficient equalization for asynchronous cooperative communications with multiple frequency offsets," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 2, pp. 648–655, 2009.
- [31] Y.-R. Tsai, H.-Y. Huang, Y.-C. Chen, and K.-J. Yang, "Simultaneous multiple carrier frequency offsets estimation for coordinated multi-point transmission in OFDM systems," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 9, pp. 4558–4568, 2013.
- [32] O. H. Salim, A. A. Nasir, H. Mehrpouyan, and W. Xiang, "Multi-relay communications in the presence of phase noise and carrier frequency offsets," *IEEE Trans. on Commun.*, vol. 65, no. 1, pp. 79–94, 2017.
- [33] W. Zhang, F. Gao, S. Jin, and H. Lin, "Frequency synchronization for uplink massive mimo systems," *IEEE Trans. on Wireless Commun.*, vol. 17, no. 1, pp. 235–249, 2018.
- [34] U. Kunnath Ganesan, R. Sarvendranath, and E. G. Larsson, "BeamSync: Over-the-air synchronization for distributed massive MIMO systems," *IEEE Trans. on Wireless Commun.*, vol. 23, no. 7, pp. 6824–6837, 2024.
- [35] S. Singh, S. Kumar, S. Majhi, U. Satija, and C. Yuen, "Blind carrier frequency offset estimation techniques for next-generation multicarrier communication systems: Challenges, comparative analysis, and future prospects," *IEEE Commun. Surveys Tuts.*, vol. 27, no. 1, pp. 1–36, 2025.
- [36] M. Hua, M. Wang, K. W. Yang, and K. J. Zou, "Analysis of the frequency offset effect on Zadoff-Chu sequence timing performance," *IEEE Trans. on Commun.*, vol. 62, no. 11, pp. 4024–4039, 2014.
- [37] 3rd Generation Partnership Project (3GPP), "5G; NR; User Equipment (UE) conformance specification; Radio transmission and reception; Part 1: Range 1 standalone," 3GPP, Technical Specification (TS) 38.521-1, Feb 2025, 3GPP TS 38.521-1 version 18.5.0 Release 18.
- [38] S. M. Kay, *Fundamentals of statistical signal processing, Volume I: estimation theory*. Prentice-Hall, Inc., 1993.