A TENSOR DECOMPOSITION TECHNIQUE FOR SOURCE LOCALIZATION FROM MULTIMODAL DATA

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ABSTRACT

This paper studies the problem of localizing a source based on different types of signals measured at different sensing locations, where propagation models of the signals are not known. A tensor observation model is proposed to arrange such multimodal data into different layers to form a 3D data array. It is proven that the vectors extracted from the least squares rank-1 approximation of the tensor under the Tucker's model are location *signature vectors* of the source, where the vectors are unimodal and their peak locations correspond to the source location. Numerical experiments demonstrate that the proposed localization method based on tensor decomposition outperforms the baseline that heuristically averages the estimates individually from different types of data.

***Index Terms*—** Source localization, tensor decomposition, matrix completion, nonparametric estimation, data fusion

1. INTRODUCTION

With the trending concept of internet-of-things (IoT), there has been a rapid development of sensor networks for various applications in industrial and civil domains. A typical IoT network may consist of different types of sensors to detect different signals, such as electromagnetic waves, acoustic signals, temperature, and ambient light. We are interested in exploiting such multimodal data to localize a source.

Source localization is challenging in many application scenarios due to harsh propagation environment or lack of infrastructure. Source localization infrastructure typically requires precise time synchronization for time-of-arrival (TOA) based ranging, propagation parameter estimation for received signal strength (RSS) based ranging, or exhaustive channel measurements for fingerprint based localization. Such requirements may not be fulfilled by a low-cost IoT network that is formed by sensors powered by batteries or energy harvesting. However, even though the primary goal of the sensor network is not for localization, the multimodal sensing data, given a large amount of it, can still be exploited to localize a source.

This paper studies source localization problems when the signal propagation models are not known, and thus, ranging information is not available. In particular, we focus on the problem of combining data from different types of sensors (for example, from RSS measurements and TOA measurements). Note that this is challenging when parametric models are *not* available as in our case, and thus, Bayesian framework cannot apply. In prior works [1, 2] for the case of single mode data (RSS measurements), a sparse matrix observation model was proposed and it was shown that by extracting a pair of dominant singular vectors from the RSS matrix, the x and y coordinates of the source can be estimated from localizing the peaks of the two vectors, respectively. However, such a strategy cannot be directly extended to the case of multimodal data. For example, a simple addition of a RSS matrix and a TOA matrix will not work, since the matrices may have different eigenstructure and the sum of them is not necessarily low rank (a required property in [1–3]).

Our Contributions: In this paper, we propose to form a tensor observation model and extract a pair of *signature vectors* from all the different types of observation matrices using tensor decomposition techniques, where the signature vectors are two *unimodal* vectors such that their peaks correspond to the source location. We develop theoretical justification that the pair of vectors extracted from the least squares rank-1 tensor approximation under the Tucker's model [4] are indeed source signature vectors, as long as for each type of signals, the energy propagation function is decreasing in distance. We demonstrate that the proposed localization based on tensor decomposition outperforms the heuristic baselines that simply averages the estimates from different types of data individually.

Relation to Prior Work: Many existing source localization techniques [5–7] require knowledge of the specific signal propagation model, which is difficult to obtained in many application scenarios. Model-free positioning schemes, such as connectivity based localizations and weighted centroid local-

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izations [8–11], may only provide coarse localization results and their performance highly depends on the choice of algorithm parameters. Machine learning techniques, such as kernel regression and support vector machines [12–14] usually arrive at solving non-linear regression problems, which suffer from too many local optima for reliable localization performance. Prior works [1, 2] proposed matrix observation models based on only RSS measurements, and advanced algorithms were developed to separate two sources in [3], and arbitrary number of sources in [15]. Preliminary work [16] proposed a heuristic method to extract the signature vector using an outer product model for tensor decomposition. More broadly, data fusion using Bayesian framework or other heuristic methods were studied in [17–19]. Yet, they still need to assume some parametric form of the propagation model.

2. SYSTEM MODEL

2.1. Signal Model

We wish to localize a source in an area with radius $L/4$. Assume that the source signals can be measured by a set of sensors that are distributed randomly and uniformly in the target area $\mathcal{A}$ with radius $L/2$. Suppose that there are $K > 1$ *modes* (*i.e.*, types) of signals that can be detected from the source. For example, an object can transmit electromagnetic waves that form an energy field and a temporal field (*i.e.*, RSS and TOA observed at different locations), and it may also emit acoustic waves due to mechanical movement.

Let $\mathcal{M}_k$ be the set of sensors that measure the kth mode signal ($\mathcal{M}_k$ and $\mathcal{M}_j$ may contain the same sensor), $d(\boldsymbol{z}, \boldsymbol{s}) \triangleq \|\boldsymbol{z} - \boldsymbol{s}\|_2$ be the distance from the source at $\boldsymbol{s} \in \mathbb{R}^2$ to the reference location $\boldsymbol{z} \in \mathbb{R}^2$, and $h_k(d)$ be the kth mode signal propagation function in distance d. The measurement $\mathsf{h}_k^{(m)}$ of the kth mode signal by the mth sensor at location $\boldsymbol{z}^{(m)}$ is given by

$$\mathsf{h}_k^{(m)} = \alpha_k h_k(d(\boldsymbol{z}^{(m)}, \boldsymbol{s})) + \mathsf{n}_k^{(m)}, \qquad m \in \mathcal{M}_k$$

where $\mathsf{n}_k^{(m)}$ is additive noise with variance σ_k^2 and is assumed to be independent across all signal modes k and all sensors m. We assume that $h_k(d)$ are *non-negative* and strictly *decreasing* functions. In addition, they are normalized and concentrate in the target area $\mathcal{A}$, *i.e.*, $\iint_{\mathbb{R}^2} h_k(d(\boldsymbol{z}, \boldsymbol{s}))^2 d\boldsymbol{z} = \iint_{\mathcal{A}} h_k(d(\boldsymbol{z}, \boldsymbol{s}))^2 d\boldsymbol{z} = 1$, and α_k^2 denotes the total energy of the kth mode signal. Note that, neither α_k nor $h_k(d)$ are known.

We discretize the $L \times L$ area that contains the target region, $\mathcal{A}$, into $N \times N$ equally spaced grid points, where the center location of the (i, j)th grid point is denoted as $\boldsymbol{c}_{i,j} \in \mathbb{R}^2$. Let $\boldsymbol{H}_k \in \mathbb{R}^{N \times N}$ be the discretized mode-k signal field matrix, where the (i, j)th entry of $\boldsymbol{H}_k$ is given by

$$H_k(i, j) = \frac{L}{N} \alpha_k h_k(d(\boldsymbol{c}_{i,j}, \boldsymbol{s})). \tag{1}$$

Let $\mathbf{H}_k \in \mathbb{R}^{N \times N}$ be a noisy and incomplete observation of $\boldsymbol{H}_k$ based on the set of measurements $\{\mathsf{h}_k^{(m)}\}$, where the (i, j)th entry of $\mathbf{H}_k$ is specified as

$$\mathsf{H}_k(i, j) = \frac{L}{N} \mathsf{h}_k^{(m)} \tag{2}$$

if the mth measurement for the kth mode signal is taken inside the (i, j)th grid.

2.2. Matrix Decomposition for Single Mode Signal

Definition 1 (Signature Vector)**.** The signature vectors of the source $\boldsymbol{s} = (s_1, s_2)$ locating inside the (m, n)th grid are defined as *unimodal* vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$, where they take maximum values at the mth and nth entries, respectively. Here, a vector $\boldsymbol{w} \in \mathbb{R}^N$ is unimodal if its entries satisfy $0 \leq w_1 \leq w_2 \leq \cdots \leq w_s \geq w_{s+1} \geq \cdots w_N \geq 0$ for some integer $1 \leq s \leq N$ and w_i is the ith entry of $\boldsymbol{w}$.

In [1, 2], it has been shown that for some classes of energy fields, the location *signature vectors* of the source can be extracted as the pair of dominant singular vectors of $\boldsymbol{H}_k$. Furthermore, one can show that there exists a symmetric function $w(x) = w(-x)$, such that $\boldsymbol{u}_{k,1}$ and $\boldsymbol{v}_{k,1}$ can be discretized from $w(x - s_1)$ and $w(x - s_2)$, respectively. Therefore, one may first apply a matrix completion algorithm to fill in the missing entries of the noisy observation matrix $\mathbf{H}_k$, then extract the dominant signature vectors $\mathbf{u}_{k,1}$ and $\mathbf{v}_{k,1}$ from the singular value decomposition (SVD) of $\mathbf{H}_k$. Finally, the source location estimate $\hat{\mathbf{s}}_k$ can be obtained by finding the peaks of $\mathbf{u}_{k,1}$ and $\mathbf{v}_{k,1}$ [3].

The fundamental problem of this paper is to combine the estimates $\hat{\mathbf{s}}_k$, $k = 1, 2, \ldots, K$, from different signals. Note that a simple average of $\hat{\mathbf{s}}_k$ may not work, because some $\mathbf{H}_k$ may have too few entries to arrive at a good estimate $\hat{\mathbf{s}}_k$ from the measurements of the kth mode signal. Although a good combination can be obtained by weighting the estimates $\hat{\mathbf{s}}_k$ by their variance, obtaining the exact variance of $\hat{\mathbf{s}}_k$ is not possible, since the propagation model $h_k(d)$ is unknown.

2.3. Tensor Model for Multimodal Signals

We propose to extract a pair of *common* signature vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ from all the observation matrices $\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_K$. The intuition is that although the $\mathbf{H}_k$ are obtained based on different propagation functions $h_k(d)$, they share the same structure where they are unimodal (for each row and column vectors) with peaks that appear at the same entry corresponding to the source location.

We use the following notation to establish a tensor model.

Tensor: An order-3 tensor , denoted as $\boldsymbol{\mathfrak{X}} \in \mathbb{R}^{N_1 \times N_2 \times N_3}$, is an array of matrices $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{N_3} \in \mathbb{R}^{N_1 \times N_2}$ arranged in such a way that the (i, j, k)th entry of $\boldsymbol{\mathfrak{X}}$, denoted

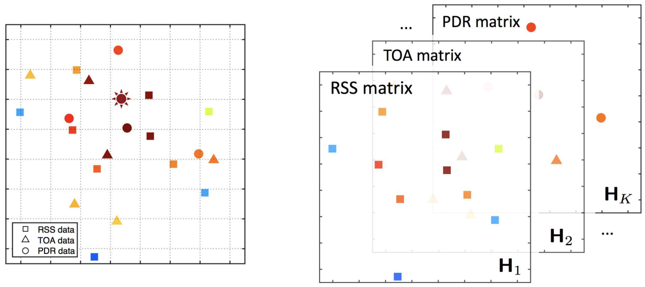

Fig. 1. (Left) Different types of data collected at different locations. The star represents the source. (Right) To arrange the data into multiple layers according to the type of data, the measurements form a sparse data cube (tensor $\boldsymbol{\mathcal{X}}$).

as $\mathfrak{X}(i, j, k)$, is given by the (i, j)th entry of $\boldsymbol{X}_k$. A geometric illustration of such a 3D data array is given in Fig. 1.

Matrix unfolding: The order-p matrix unfolding, denoted as $\boldsymbol{X}_{(p)}$, of a tensor $\boldsymbol{\mathfrak{X}}$ generated from matrices $\boldsymbol{X}_1, \boldsymbol{X}_2, \dots, \boldsymbol{X}_{N_3}$ are defined as

$$\boldsymbol{X}_{(1)} = [\boldsymbol{X}_1, \boldsymbol{X}_2, \dots, \boldsymbol{X}_{N_3}]^{\mathrm{T}} \in \mathbb{R}^{N_2 N_3 \times N_1}$$
$$\boldsymbol{X}_{(2)} = [\boldsymbol{X}_1^{\mathrm{T}}, \boldsymbol{X}_2^{\mathrm{T}}, \dots, \boldsymbol{X}_{N_3}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{N_1 N_3 \times N_2}$$
$$\boldsymbol{X}_{(3)} = [\mathrm{vec}(\boldsymbol{X}_1), \mathrm{vec}(\boldsymbol{X}_2), \dots, \mathrm{vec}(\boldsymbol{X}_{N_3})] \in \mathbb{R}^{N_1 N_2 \times N_3}$$

where $\mathrm{vec}(\boldsymbol{X}) = [\boldsymbol{x}_1^{\mathrm{T}}, \boldsymbol{x}_2^{\mathrm{T}}, \dots, \boldsymbol{x}_N^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{x}_i$ is the ith column of $\boldsymbol{X}$.

In this paper, we use $\boldsymbol{\mathfrak{X}} \in \mathbb{R}^{N \times N \times K}$ to denote the underlying signal field described by matrices $\boldsymbol{H}_1, \boldsymbol{H}_2, \dots, \boldsymbol{H}_K$ defined in (1), whereas, the noisy observation of $\boldsymbol{\mathfrak{X}}$, denoted as $\boldsymbol{\mathcal{X}}$, is an array of (incomplete) matrices $\mathsf{H}_1, \mathsf{H}_2, \dots, \mathsf{H}_K$ defined in (2).

3. COMMON SIGNATURE VECTORS FROM RANK-1 TENSOR APPROXIMATION

We conjecture that the vectors obtained from the least squares rank-1 approximation of $\boldsymbol{\mathfrak{X}}$ are also signature vectors. Note that this is not obvious, since there exist many decomposition models for a tensor. Herein, we prove this conjecture.

To formulate the problem, we use the form-p tensor-matrix multiplication (also known as mode-p multiplication) defined in [20]. Essentially, multiplying a $N_1 \times N_2 \times \cdots \times N_P$ tensor $\boldsymbol{\mathfrak{X}}$ by a $M \times N_p$ matrix $\boldsymbol{A}$, denoted as $\boldsymbol{\mathfrak{X}} \times_p \boldsymbol{A}$, yields a $N_1 \times N_2 \times \cdots N_{p-1} \times M \times N_{p+1} \times \cdots \times N_P$ tensor, with its $(i_1, i_2, \cdots, i_{p-1}, m, i_{p+1}, \cdots, i_P)$th entry given by $\sum_{i_p=1}^{N_p} \mathfrak{X}(i_1, i_2, \cdots, i_{p-1}, i_p, i_{p+1}, \cdots, i_P) A_{m,i_p}$, where A_{m,i_p} is the (m, i_p)th entry of $\boldsymbol{A}$. In the special case for a $N_1 \times N_2$ tensor (*i.e.*, matrix) $\boldsymbol{\mathfrak{X}}$, we have $\boldsymbol{\mathfrak{X}} \times_1 \boldsymbol{A} \times_2 \boldsymbol{B} = \boldsymbol{A}\boldsymbol{\mathfrak{X}}\boldsymbol{B}^{\mathrm{T}}$. Moreover, one can easily verify that $\boldsymbol{\mathfrak{X}} \times_p \boldsymbol{A} \times_q \boldsymbol{B} = \boldsymbol{\mathfrak{X}} \times_q \boldsymbol{B} \times_p \boldsymbol{A}$ for $p \neq q$.

Using this notation, we define the least squares rank-1 approximation of tensor $\boldsymbol{\mathfrak{X}} \in \mathbb{R}^{N \times N \times K}$ as the vectors $\boldsymbol{w}_1, \boldsymbol{w}_2 \in \mathbb{R}^N$ and $\boldsymbol{w}_3 \in \mathbb{R}^K$ that solve the following problem

$$\mathscr{P}_0: \quad \underset{\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \alpha}{\text{minimize}} \quad \|\boldsymbol{\mathfrak{X}} - \alpha \times_1 \boldsymbol{w}_1 \times_2 \boldsymbol{w}_2 \times_3 \boldsymbol{w}_3\|_{\mathrm{F}}^2$$
$$\text{subject to} \quad \alpha > 0, \|\boldsymbol{w}_1\| = \|\boldsymbol{w}_2\| = \|\boldsymbol{w}_3\| = 1$$

where $\|\boldsymbol{\mathfrak{X}}\|_{\mathrm{F}}^2 \triangleq \sum_i \sum_j \sum_k \mathfrak{X}(i, j, k)^2$ for an order-3 tensor and $\|\boldsymbol{w}\|$ is the Euclidean norm for a vector.

Note that when $K = 1$, problem $\mathscr{P}_0$ degenerates to a matrix SVD problem since $\alpha \times_1 \boldsymbol{w}_1 \times_2 \boldsymbol{w}_2 = \alpha \boldsymbol{w}_1 \boldsymbol{w}_2^{\mathrm{T}}$, where the solutions $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are given by the dominant singular vectors of $\boldsymbol{\mathfrak{X}}$ (as a matrix) and $\boldsymbol{w}_3 = 1$ will be a scalar. However, in contrast to the matrix case, solving the global optimal solution to $\mathscr{P}_0$ is, in general, NP-hard [21]. It is also not known whether the solution preserves the desired unimodal property.

3.1. Unimodal Property

We find that the answer is affirmative: unimodality is preserved.

Theorem 1 (Tensor Unimodality). *The optimal solutions $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ to problem $\mathscr{P}_0$ are unimodal. In addition, if the source $\boldsymbol{s}$ locates inside the (m, n)th grid, then, the peaks of $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ locate at the mth entry of $\boldsymbol{w}_1$ and the nth entry of $\boldsymbol{w}_2$, respectively.*

Proof. (Sketch). It can be shown that solving $\mathscr{P}_0$ is equivalent to maximizing $\|\boldsymbol{\mathfrak{X}} \times_1 \boldsymbol{w}_1^{\mathrm{T}} \times_2 \boldsymbol{w}_2^{\mathrm{T}} \times_3 \boldsymbol{w}_3^{\mathrm{T}}\|_{\mathrm{F}}^2$ under the same set of constraints [20].

One can easily verify that

$$\begin{aligned} f(\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3) &\triangleq \|\boldsymbol{\mathfrak{X}} \times_1 \boldsymbol{w}_1^{\mathrm{T}} \times_2 \boldsymbol{w}_2^{\mathrm{T}} \times_3 \boldsymbol{w}_3^{\mathrm{T}}\|_{\mathrm{F}} \\ &= |\boldsymbol{w}_1^{\mathrm{T}} (\boldsymbol{\mathfrak{X}} \times_3 \boldsymbol{w}_3^{\mathrm{T}}) \boldsymbol{w}_2| \\ &= |\boldsymbol{w}_1^{\mathrm{T}} (\sum_{k=1}^{K} w_{3,k} \boldsymbol{H}_k) \boldsymbol{w}_2| \end{aligned}$$

where $w_{3,k}$ is the kth entry of $\boldsymbol{w}_3$. It is clear that given $\boldsymbol{w}_3$, the maximum value of f is given by the dominant singular value of $\tilde{\boldsymbol{X}}^{(3)} \triangleq \sum_{k=1}^{K} w_{3,k} \boldsymbol{H}_k$. It can also be shown that the optimal solution $\boldsymbol{w}_3$ must not contain negative entries (except that all the entries of $\boldsymbol{w}_3$ are negative), as the entries of $\boldsymbol{H}_k$ are non-negative due to (1). On the other hand, the optimal solutions $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ that maximize f are the dominant singular vectors of $\tilde{\boldsymbol{X}}^{(3)}$.

Therefore, it can be further verified that the dominant right singular vector of $\tilde{\boldsymbol{X}}^{(3)}$ is unimodal. First, the matrix $\boldsymbol{R} \triangleq \tilde{\boldsymbol{X}}^{(3)\mathrm{T}} \tilde{\boldsymbol{X}}^{(3)}$ is unimodal since the rows of $\tilde{\boldsymbol{X}}^{(3)}$ are unimodal [22, Lemma 2], and so is the matrix $\boldsymbol{R}^q / \mathrm{tr}\{\boldsymbol{R}^q\}$ [22, Lemma 3]. Then, in the limit, $\boldsymbol{R}^\infty / \mathrm{tr}\{\boldsymbol{R}^\infty\}$ is rank-1 and unimodal, and as a result, its eigenvector is unimodal, which means that the dominant right singular vector of $\tilde{\boldsymbol{X}}^{(3)}$ is unimodal. This justifies that $\boldsymbol{w}_2$ is unimodal. Similar arguments apply to $\boldsymbol{w}_1$. $\square$

Algorithm 1 Signature vector from a tensor model

1. Initialize $\boldsymbol{\mathcal{Y}}$ from $\boldsymbol{\mathcal{X}}$ (*e.g.*, via matrix completion for each component $\mathbf{H}_k$ [1, 2]).
2. Initialize $\boldsymbol{w}_1$, $\boldsymbol{w}_2$, and $\boldsymbol{w}_3$ by the right dominant singular vectors of $\boldsymbol{Y}_{(1)}$, $\boldsymbol{Y}_{(2)}$, and $\boldsymbol{Y}_{(3)}$, respectively, where $\boldsymbol{Y}_{(p)}$ are the order-p matrix unfolding of $\boldsymbol{\mathcal{Y}}$.
3. Update sequentially: $\boldsymbol{w}_1 \leftarrow (\boldsymbol{w}_3 \otimes \boldsymbol{w}_2)^{\mathrm{T}}\boldsymbol{Y}_{(1)}/\|(\boldsymbol{w}_3 \otimes \boldsymbol{w}_2)^{\mathrm{T}}\boldsymbol{Y}_{(1)}\|$, $\boldsymbol{w}_2 \leftarrow (\boldsymbol{w}_3 \otimes \boldsymbol{w}_1)^{\mathrm{T}}\boldsymbol{Y}_{(2)}/\|(\boldsymbol{w}_3 \otimes \boldsymbol{w}_1)^{\mathrm{T}}\boldsymbol{Y}_{(2)}\|$, $\boldsymbol{w}_3 \leftarrow (\boldsymbol{w}_2 \otimes \boldsymbol{w}_1)^{\mathrm{T}}\boldsymbol{Y}_{(3)}$, $\alpha = \|\boldsymbol{w}_3\|$, $\boldsymbol{w}_3 \leftarrow \boldsymbol{w}_3/\alpha$, where $\otimes$ denotes the Kronecker product.
4. Update $\boldsymbol{\mathcal{Y}}$ as the solution that minimizes $\|\boldsymbol{\mathcal{Y}} - \alpha \times_1 \boldsymbol{w}_1 \times_2 \boldsymbol{w}_2 \times_3 \boldsymbol{w}_3\|_{\mathrm{F}}^2$ subject to $\|\mathcal{P}_\Omega\{\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{X}}\}\|_{\mathrm{F}} \leq \epsilon$.
5. Repeat from Step 3 until convergence.

3.2. Signature Vector from Noisy and Incomplete Observations

Given the noisy and incomplete observation $\boldsymbol{\mathcal{X}}$, one can extract the signature vectors by solving the tensor completion problem

$$\mathscr{P}_1: \quad \underset{\boldsymbol{\mathcal{Y}}, \boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \alpha}{\text{minimize}} \quad \|\boldsymbol{\mathcal{Y}} - \alpha \times_1 \boldsymbol{w}_1 \times_2 \boldsymbol{w}_2 \times_3 \boldsymbol{w}_3\|_{\mathrm{F}}^2$$
$$\text{subject to} \quad \|\mathcal{P}_\Omega\{\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{X}}\}\|_{\mathrm{F}} \leq \epsilon$$
$$\alpha > 0, \|\boldsymbol{w}_1\| = \|\boldsymbol{w}_2\| = \|\boldsymbol{w}_3\| = 1$$

where Ω is the set of observed entries, $[\mathcal{P}_\Omega\{\boldsymbol{\mathcal{X}}\}]_{ijk} = x_{ijk}$ if $(i, j, k) \in \Omega$ is observed, and $[\mathcal{P}_\Omega\{\boldsymbol{\mathcal{X}}\}]_{ijk} = 0$ otherwise.

Problem $\mathscr{P}_1$ can be solved by a block coordinate descent algorithm [23] as summarized in Algorithm 1.

In the special case of complete and noiseless observations, $\mathcal{P}_\Omega\{\boldsymbol{\mathcal{X}}\} = \mathfrak{X}$, problem $\mathscr{P}$ degenerates to a tensor decomposition problem. One can show the convergent vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ obtained from Algorithm 1 are unimodal (although they may not be the global optimal solution to $\mathscr{P}_1$, which is a NP-hard problem [21]). To see this, one can easily verify that $\boldsymbol{w}_1$ are $\boldsymbol{w}_2$ are unimodal after the initialization step and after each iteration.

4. NUMERICAL RESULTS

We consider the source and sensor deployment model in Section 2 with $L = 200$ meters. Half of the sensors detect the RSS of the electromagnetic signal and the other sensors detect the TOA from the acoustic signal. The noise-normalized RSS signal is modeled as $P_{\mathrm{dB}}(d) = 70 - 36 \times \log_{10}(\max\{10, d\}) + S$, where $S \sim \mathcal{N}(0, \sigma_{\mathrm{s}}^2)$ is to model log-normal shadowing and $\sigma_{\mathrm{s}} = 10$ dB. The TOA signal is modeled as $t(d) = d/c + b$, where $c = 340$ m/s and $b \sim \mathcal{N}(0, \sigma_{\mathrm{t}}^2)$ is to model synchronization errors and $\sigma_{\mathrm{t}} = 100$ ms. To normalize the data, we use $h_1(d) = \exp(-\beta_1 10^{-P_{\mathrm{dB}}(d)/10})$ and $h_2(d) = \exp(-\beta_2 t(d)^2)$, where the parameters β_1 and β_2 are chosen based on the raw measurements P_{dB} and t such that the normalized measurement data $\{\mathsf{h}_1^{(m)} : m \in \mathcal{M}_1\}$ and $\{\mathsf{h}_2^{(m)} : m \in \mathcal{M}_2\}$ is roughly uniformly distributed over $(0, 1)$. The dimension parameter N is chosen as the largest integer satisfying $1.5N(\log N)^2 \leq \sum_k |\mathcal{M}_k|$ [4]. The peak localization algorithm for unimodal vectors in [3] is used for all schemes to estimate the source location.

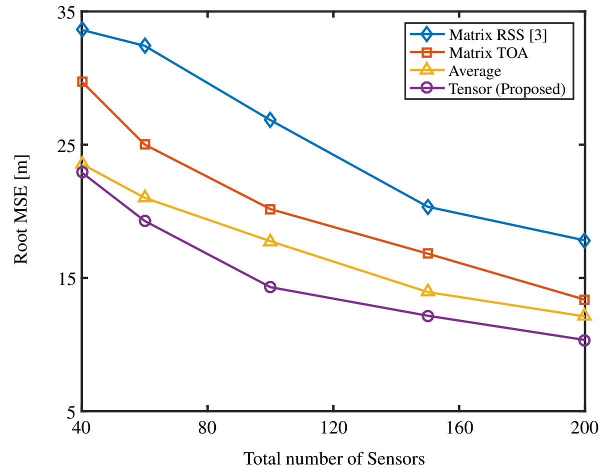

Fig. 2. Root MSE versus the total number of sensors.

Fig. 2 shows the localization performance in root mean squared error (MSE) versus the total number of sensors. The "Matrix" schemes process the RSS and TOA data individually, using the matrix observation model to extract the source signature vector followed by peak localization to estimate the source location [3]. The "Average" scheme simply takes an average from the RSS and TOA based estimation $\hat{\mathbf{s}}_{\mathrm{avg}} = (\hat{\mathbf{s}}_{\mathrm{RSS}} + \hat{\mathbf{s}}_{\mathrm{TOA}})/2$. The proposed scheme extracts common source signature vectors from the RSS and TOA data using Algorithm 1. It performs the best over all the schemes, because it exploits the common unimodal structure from the RSS and TOA data.

5. CONCLUSIONS

This paper developed a sparse tensor decomposition method to extract the location signature vectors for source localization from multimodal data. A tensor observation model was proposed to arrange different types of measurements into different layers. It is shown that the vectors extracted from the least squares rank-1 tensor approximation have a unimodal property, where the peak locations of the vectors correspond to the source location in x and y coordinates, respectively. Based on such a theoretical guarantee, it is demonstrated that the tensor method outperforms the baselines that simply average the estimates from different types of data individually.

6. REFERENCES

[1] S. Choudhary and U. Mitra, "Analysis of target detection via matrix completion," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, 2015, pp. 3771–3775.

[2] S. Choudhary, N. Kumar, S. Narayanan, and U. Mitra, "Active target localization using low-rank matrix completion and unimodal regression," *arXiv preprint arXiv:1601.07254*, 2016.

[3] J. Chen and U. Mitra, "Rotated eigenstructure analysis for source localization without energy-decay models," in *Proc. Int. Conf. Digital Signal Process.*, London, UK, Aug. 2017.

[4] N. D. Sidiropoulos, L. De Lathauwer, X. Fu, K. Huang, E. E. Papalexakis, and C. Faloutsos, "Tensor decomposition for signal processing and machine learning," *IEEE Trans. Signal Process.*, vol. 65, no. 13, pp. 3551–3582, 2017.

[5] A. Beck, P. Stoica, and J. Li, "Exact and approximate solutions of source localization problems," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1770–1778, 2008.

[6] C. Meesookho, U. Mitra, and S. Narayanan, "On energy-based acoustic source localization for sensor networks," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 365–377, 2008.

[7] Y. Liu, Y. H. Hu, and Q. Pan, "Distributed, robust acoustic source localization in a wireless sensor network," *IEEE Trans. Signal Process.*, vol. 60, no. 8, pp. 4350–4359, 2012.

[8] H. Chen, Q. Shi, R. Tan, H. V. Poor, and K. Sezaki, "Mobile element assisted cooperative localization for wireless sensor networks with obstacles," *IEEE Trans. Wireless Commun.*, vol. 9, no. 3, 2010.

[9] T. He, C. Huang, B. M. Blum, J. A. Stankovic, and T. Abdelzaher, "Range-free localization schemes for large scale sensor networks," in *Proc. Int. Conf. Mobile Computing and Networking*, 2003, pp. 81–95.

[10] J. Blumenthal, R. Grossmann, F. Golatowski, and D. Timmermann, "Weighted centroid localization in Zigbee-based sensor networks," in *Prof. IEEE Int. Symp. Intelligent Signal Process.*, 2007, pp. 1–6.

[11] J. Wang, P. Urriza, Y. Han, and D. Cabric, "Weighted centroid localization algorithm: theoretical analysis and distributed implementation," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3403–3413, 2011.

[12] X. Nguyen, M. I. Jordan, and B. Sinopoli, "A kernel-based learning approach to ad hoc sensor network localization," *ACM Trans. on Sensor Networks*, vol. 1, no. 1, pp. 134–152, 2005.

[13] Y. Jin, W.-S. Soh, and W.-C. Wong, "Indoor localization with channel impulse response based fingerprint and nonparametric regression," *IEEE Trans. Wireless Commun.*, vol. 9, no. 3, pp. 1120–1127, 2010.

[14] W. Kim, J. Park, J. Yoo, H. J. Kim, and C. G. Park, "Target localization using ensemble support vector regression in wireless sensor networks," *IEEE Trans. on Cybernetics*, vol. 43, no. 4, pp. 1189–1198, 2013.

[15] J. Chen and U. Mitra, "Underwater acoustic source localization using unimodal-constrained matrix factorization," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, CA, USA, Nov. 2017.

[16] S. Honnungar, S. Choudhary, and U. Mitra, "On target localization with communication costs via tensor completion: A multi-modal approach," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, 2016, pp. 6400–6404.

[17] J. Prieto, S. Mazuelas, A. Bahillo, P. Fernandez, R. M. Lorenzo, and E. J. Abril, "Adaptive data fusion for wireless localization in harsh environments," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1585–1596, 2012.

[18] Y. Li, M.-H. Meng, and W. Chen, "Data fusion based on RBF and nonparametric estimation for localization in wireless sensor networks," in *Proc. IEEE Int. Conf. Robotics and Biomimetics*, 2007, pp. 1361–1365.

[19] A. T. Ihler, J. W. Fisher, R. L. Moses, and A. S. Willsky, "Nonparametric belief propagation for self-localization of sensor networks," vol. 23, no. 4, pp. 809–819, 2005.

[20] L. De Lathauwer, B. De Moor, and J. Vandewalle, "On the best rank-1 and rank-(r1, r2,..., rn) approximation of higher-order tensors," *SIAM J. on Matrix Analysis and Applications*, vol. 21, no. 4, pp. 1324–1342, 2000.

[21] C. J. Hillar and L.-H. Lim, "Most tensor problems are NP-hard," *Journal of the ACM*, vol. 60, no. 6, p. 45, 2013.

[22] J. Chen and U. Mitra, "Matrix factorization for nonparametric multi-source localization exploiting unimodal properties," 2017, submitted to *IEEE Trans. Signal Process.*, preprint arXiv:1711.07457.

[23] Y. Xu and W. Yin, "A block coordinate descent method for regularized multiconvex optimization with applications to nonnegative tensor factorization and completion," *SIAM J. on imaging sciences*, vol. 6, no. 3, pp. 1758–1789, 2013.